International University Bremen, Fall 2006
School of Engineering and Science
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Advanced Physics A I

Problem sheet 4

due Thursday, 2006.10.12

Problem 12 (5 points)

Consider a mass point moving along an ellipse in the plane with a frequency such that it needs two seconds for three full orbits (i.e. for 6π).

a) Derive the trajectory in Cartesian coordinates

$$\vec{r}(t) = (x(t), y(t), z(t))$$

where $x(t) = a\cos(\omega t)$ **(1 point)**

b) What is the force acting on the mass point? **(1 point)**

c) Calculate the angular momentum of the mass point. Why is it conserved? **(1 point)**

d) What is the area ΔA swept out by the position vector in one second? **(2 points)**

Problem 13 (13 points)

Consider a particle of mass m in a force field with potential:

$$V(r) = \frac{\alpha}{r^2}$$

a) What do you know about force, energy, and angular momentum from the general considerations in the lecture? **(2 points)**

b) Choose the coordinate system such that for $\alpha > 0$:

$$r_{min} = r(t = 0), \phi(r_{min}) = 0$$

Note that the radial velocity vanishes at r_{min}. Plot V_{eff} qualitatively if it is defined in analogy to the lecture. Calculate r_{min} as a function of l and E. **(3 points)**

c) Determine $r = r(t)$ and $r = r(\phi)$ for $E > 0$ and $\alpha > 0$. What is the trajectory for $\alpha > 0$? **(3 points)**
Hint: Express E and $\dot{r}$ in terms of r_{min}. For solving the integral replace $r_{min}/r' \equiv z$. Calculate first $t(r)$, next $r(t)$; use the angular momentum to find $\phi(r)$ and then $r(\phi)$.

d) **Voluntary** How does V_{eff} change for $\alpha < 0$? When do we have a bounded motion for an attractive potential ($\alpha < 0$)? Determine r_{max} for this case. **(3 points)**

e) **Voluntary** For $r(t = 0) = r_{max}$ determine the time t_0 after which the particle ends up in the center of the force field at $r = 0$. **(2 points)**

Problem 14 (6 points)

There is a famous so-called Lenz-Runge vector that can be derived as a conserved quantity for $1/r$-potentials. It is defined according to:

$$\vec{A} = (\dot{\vec{r}} \times \vec{L}) + V(r)\vec{r}$$

with $\vec{L}$ the angular momentum and $V(r)$ the potential. $\vec{A}$ is the Lenz-Runge vector.

a) Let $V(r) = -\frac{\alpha}{r}$ for $\alpha > 0$. Show that $\vec{A}$ is a conserved quantity. **(2 points)**

b) What is the absolute value of $\vec{A}$? Use that the velocity is perpendicular to the angular momentum. **(2 points)**

c) **voluntary** Write the trajectory $r(\phi)$ by means of $\vec{A}$ in the form:

$$\frac{1}{r} = \frac{1 + \epsilon \cos\phi}{const},$$

where $\phi = \angle(\vec{A}, \vec{r})$, and *const* as well as ϵ are expressed in terms of m, α, E, and l. **(2 points)**
Hint: Start with an evaluation of $\vec{A} \cdot \vec{r}$.