SOLUTION KEY MATH 255 QUIZ I

Computer Eng.

1. Find the particular solution of the following equation.

$$y' + \frac{2}{x} y = \frac{e^x}{x}, \quad y(1) = 0$$

$$P(x) = \frac{2}{x} \qquad \mu(x) = e^{\int P(x)dx} = e^{\int \frac{2}{x} dx} = e^{\ln x^2} = x^2$$

$$dy + \frac{2}{x} y\, dx = \frac{e^x}{x} dx$$

$$dy + \left(\frac{2}{x} y - \frac{e^x}{x}\right) dx = 0$$

$$dy + \frac{1}{x}(2y - e^x)\, dx = 0 \rightarrow \text{multiply with } x^2$$

$$\underbrace{x^2}_{N} dy + \underbrace{x(2y - e^x)}_{M} dx = 0$$

$$\frac{\partial M}{\partial y} = 2x = \frac{\partial N}{\partial x} = 2x \qquad \text{Exact Diff.}$$

$$dF = \frac{\partial F}{\partial y} dy + \frac{\partial F}{\partial x} dx = 0$$

$$F = \int x^2 dy = x^2 y + g(x)$$

$$\frac{\partial F}{\partial x} = \cancel{2xy} + \frac{dg}{dx} = \cancel{2xy} - xe^x \rightarrow g = \int -x e^x\, dx = xe^x - e^x + C$$

$$\left.\begin{matrix} x & e^x & \\ 1 & & e^x \\ 0 & & e^x \end{matrix}\right\} \text{using tabular}$$

$$F = x^2 y + xe^x - e^x = C$$

$$y(1) = 0 \qquad 1.0 + 1.\cancel{e} - \cancel{e} = C \rightarrow \boxed{C = 0}$$

$$\boxed{x^2 y + xe^x - e^x = 0}$$

2. $$y' = \frac{e^x}{y^2 - 1} \rightarrow \frac{dy}{dx} = \frac{e^x}{y^2 - 1} \rightarrow \int (y^2 - 1) dy = \int e^x dx$$

Separation of variables

$$\frac{y^3}{3} - y = e^x + C \quad \text{↳ constant}$$

$$\boxed{3y = 3e^x - y^3}$$

3. a) $y^{(IV)} + \sin y\, y'' + xy = e^x$ → nonlinear, ordinary and 4th order diff. eq.

b) $\frac{dy}{dx} + e^x y = y^3$ → non linear, ordinary, 1st order diff. eqn.

c) $\frac{\partial^2 u}{\partial x^2} - \frac{\partial^2 u}{\partial y^2} = 1$ linear, partial, 2nd order diff. eqn.