Qu. 2.1

$$\text{so} \quad \hat{p} \cdot e^{i\underline{k}\cdot\underline{r}} = (-i\hbar\vec{\nabla})\, e^{i\underline{k}\cdot\underline{r}}$$

$$= (-i\hbar)(i\vec{k})\, e^{i\underline{k}\cdot\underline{r}}$$

$$= (\hbar\vec{k})\, e^{i\underline{k}\cdot\underline{r}}$$

For future use, note that the operator

$$\hat{T}_R = e^{-i\underline{\hat{p}}\cdot\underline{R}/\hbar}$$

generates translations:

$$\text{ie} \quad \hat{T}_R \cdot f(\underline{r}) = f(\underline{r}+\underline{R}).$$

Proof:

$$\hat{T}_R \cdot f(r) = e^{\underline{R}\cdot\underline{\nabla}} f(r)$$

$$= \left[1 + \underline{R}\cdot\underline{\nabla} + \frac{1}{2}(\underline{R}\cdot\underline{\nabla})^2 \ldots\right.$$

$$= f(r) + \underline{R}\cdot\nabla f(r) \ldots$$

ϵ (Taylor series expansion of $f(r)$)

$$= f(r+R).$$