Math 102 Homework Assignment 8. Due Thursday March 13 by 4pm

1. Show that the set of 2×3 matrices of the form $\begin{bmatrix} a & b & 0 \\ c & 0 & d \end{bmatrix}$ forms a subspace of the vector space of all 2×3 matrices.

2. Let V be the set of all polynomials $ax^3 + bx^2 + cx + d$ that satisfy the condition $a + b + c + d = 0$. Determine whether or not V is a subspace of P_3.

3. Let

$$V = \left\{ \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}, \begin{bmatrix} -1 \\ 0 \\ 4 \end{bmatrix}, \begin{bmatrix} 5 \\ -1 \\ 2 \end{bmatrix} \right\}.$$

Determine whether V is linearly dependent or linearly independent.

4. Find all values of k so that the set of vectors $\left\{ \begin{bmatrix} 0 \\ 1 \\ k \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} -k \\ 0 \\ 1 \end{bmatrix} \right\}$ spans $\mathbb{R}^3$.

5. Determine a basis for and the dimension of the solution space of the following homogeneous system:

$$\begin{aligned} x_1 + x_2 + x_3 + x_5 &= 0 \\ 2x_1 - 3x_2 - x_4 &= 0 \\ 3x_1 - 2x_2 + 4x_3 - x_4 + x_5 &= 0 \end{aligned}$$

6. Let $V = \text{span}\left\{ \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}, \begin{bmatrix} 5 & 4 \\ 0 & -1 \end{bmatrix}, \begin{bmatrix} 1 & 2 \\ 0 & 2 \end{bmatrix}, \begin{bmatrix} 2 & 1 \\ 0 & 0 \end{bmatrix} \right\}$. Find a basis for V.