a monograph on sediment transport in alluvial streams

by

frank engelund and eggert hansen

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Technical University of Denmark
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PREFACE.

The present short text on sediment transport in alluvial streams does not pretend to give a complete review on the present state of knowledge. Neither has it been the purpose to account for the historical development of this particular branch of hydraulic science. Consequently, readers familiar with the subject will probably find, that many important findings in recent years have not been included or have only been mentioned superficially.

The immediate reason for writing has been a need for a short textbook as a basis for lectures to be given at the Technical University of Denmark for students specializing in hydraulic or coastal engineering. Since this audience is very small it was decided to publish the lectures in English.

As the general understanding of the basic mechanism of sediment transport is at its beginning, it has been the aim of the authors to avoid all mathematical sophistication, so that the presentation is based on simple "engineering methods" only. Although a short account of basic hydraulics is included to ensure continuity, the text requires knowledge of elementary fluid mechanics.

Central parts of the book is based on research carried out at the Hydraulic Laboratory in Copenhagen. Some of the results have previously been given only in discussions or in Progress Reports.

Frank Engelund and Eggert Hansen.

Technical University of Denmark,
Hydraulic Laboratory.
January 1967.

CONTENTS

List of symbols

A = area of cross section

a = migration velocity of sand waves

B = water surface width

c = volume concentration

c_D = drag coefficient

D = mean depth

d = mean fall diameter

$\mathbf{F}$ = $V/\sqrt{g\,D}$ = Froude number

f = friction factor

g = acceleration of gravity

H = energy level

ΔH = loss of energy level

h = dune height

I = energy gradient (slope)

k = surface roughness ($\sim 2.5\,d$)

L = Meander (wave-)length

l = length of dunes

n = porosity

Q = water discharge

q = ($Q/B = VD$) water discharge per unit width

Q_T = total sediment discharge (= $Q_B + Q_s$), volume of grains per second

q_T = Q_T/B

Q_B = discharge of bed load

Q_s = discharge of suspended load

$\mathbf{R}$ = $U_f\, d/\nu$ = Reynolds' number

s = γ_s/γ = relative density of sediment grains

t = time

U = local flow velocity

U_f = $\sqrt{\tau_0/\rho}$ = friction velocity

V = mean velocity

w = settling velocity

x, y, z = coordinates

γ = specific gravity of water

γ_s = specific gravity of sediment grains

δ = thickness of boundary layer or viscous sublayer

Φ = $q_T / \sqrt{(s-1)\, g\, d^3}$

θ = $D\, I / (s-1) d$

ρ = density of water

τ_0 = $\tau' + \tau''$ = total bed shear stress

τ_c = critical bed shear stress

1. INTRODUCTION.

The theory of sediment transport induced by fluid flow is one of the branches of modern hydraulic science in which a very intense research is going on all over the world. This comprehensive interest is well justified by the numerous technical applications of the subject and the obvious significance of the results in adjacent sciences such as fluviology. Many civil-engineering works in the field of flood control, canals for irrigation, reclamation, navigation and water supply is profoundly affected by moving sediments, so that a reasonably accurate theory of sediment transport is a necessary basis for a satisfactory design.

The main sources of sediment in natural streams are erosion by overland flow, stream-channel erosion, bank cutting and supply from small erosion channels formed in unconsolidated soil.

The term "alluvial" is usually applied to streams in which the moving sediment and the sediment in the underlying bed is of the same character. However, most natural streams carry a certain amount of very fine particles, the so-called wash load, that is not (or practically not) represented in the bed. Consequently, the knowledge of bed material composition does not permit any prediction of the rate of wash load transportation. Fortunately, the wash load is of secondary importance in most technical problems. When the term "total sediment discharge" is applied in the following the wash load is neglected.

One of the main difficulties of our subject is that the bed configuration depends on the discharge. Moreover, the hydraulic resistance is a complicated function of the bed configuration, so that we are faced with a very intricate problem of mutual interaction. As the problem ofsand wave formation depends on hydrodynamic stability, we even find that changes in bed formation may have a nearly discontinuous character at a certain stage (shift from the lower to the upper flow regime).

But perhaps the most important obstacle to the development of a really satisfactory sediment transport theory is that it presupposes knowledge of the mutual interaction of suspended particles and the turbulence required to agitate the particle motion. Although such a theory seems very remote, remarkable progress has been achieved by the works of H. A. Einstein and R. A. Bagnold, briefly mentioned in section 3.

Instead of pretending a mathematical description of the transport mechanism in detail, the authors have tried to give a consistent survey of the macro-features of the problem by the application of a general principle of similarity. Although the limits of validity of this principle is not known exactly to-day, it was found to be a useful working hypothesis giving a reasonably accurate description of the sediment discharge as well as of the hydraulic resistance in the most important case of dune-covered bed.

2. SEDIMENT PROPERTIES.

2.1. General remarks.

The sediment carried by natural streams contains usually particles ranging from gravel or sand down to very small particles classified as silt or clay. The very fine fractions are carried as wash load or even as colloidal suspension for which electrochemical forces play a predominant role. It should be mentioned that flocculation of colloidal particles may occur at the transition between fresh and salt water.

For the hydraulics of alluvial streams the most significant particles are those in the range of sand (0.06 mm to 2 mm) and gravel (2 mm to 20 mm), and the following short account of sediment properties is devoted to these fractions, exclusively. From a hydraulic point of view the most important sediment properties are related to size, shape and specific gravity.

2.2. Particle size characteristics.

The most usual and convenient method for the analysis of particle size distribution is the sieve analysis, which is applicable for particle sizes not smaller than 0.06 mm. An adequate number of representative sediment samples are analysed, and the result is presented as a frequency curve (figure 2. 2. 1) or as a cumulative frequency curve (distribution curve, figure 2. 2. 2).

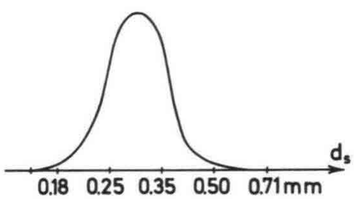

Figure 2.2.1

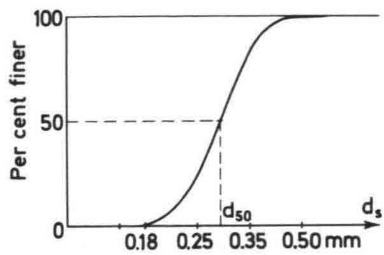

Figure 2.2.2

The frequency curve in figure 2.2. 1 is based on a set of sieves with meshes in geometrical progression, the abscissa representing the so called sieve diameter d_s and the ordinate the concentration of the total sample contained in the corresponding intervals of the d_s. Very often the distribution curve of sediments approaches the normal probability curve when plotted as in figure 2. 2. 1, so that the distribution function is log-normal.

The cumulative (or grain-size) curve in figure 2. 2. 2 is the most usual graphical representation of the grain size analysis. In this the ordinate indicates how many per cent (by weight) of the total sample are finer than the diameter d_s of the abscissa.

The mean diameter is the median, i. e. d_s corresponding to 50 pct being finer, and it is usually denoted by d_{50}. In hydraulic theory the fractiles d_{35} and d_{65} have often been utilized to characterize bed load diameter and surface roughness, respectively.

Another measure of particle size is the spherical diameter d_v, defined as the diameter of a sphere having the same volume as the given particle. In practice d_v is determined by weighing a counted number of particles from a certain fraction of the sample.

It has been objected that neither the sieve diameter d_s nor the spherical diameter d_v takes any account of the shape of the sediment grains. In this respect we may find the fall or sedimentation diameter d_f more satisfactory.

The fall diameter of a particle is defined as the diameter of a sphere having the same settling velocity in water at $24^{o}C$. For a fixed particle volume d_f will be greater for angular grains than for rounded grains, so that this size measure takes some account of the shape, refer section 3 on roughness elements.

In table 2.2.1 are listed a number of simultaneous values of d_s, d_v and d_f for some typical fractions of natural sand. The ratio between d_v and d_s is nearly constant, which is very natural when the shape of the grains is not too much dependent on the grain size. The fall diameter d_f, however, becomes significantly smaller than d_s and d_v for the larger grains.

Table 2.2.1.

Values of d_s, d_v, d_f and w for typical sand fractions.

d_s mm	d_v mm	d_f mm	$w(10^{o}C)$ m/sec	$w(20^{o}C)$ m/sec
0.089	0.10	0.10	0.005	0.007
0.126	0.14	0.14	0.010	0.013
0.147	0.17	0.16	0.013	0.016
0.208	0.22	0.22	0.023	0.028
0.25	0.25	0.25	0.028	0.033
0.29	0.30	0.29	0.033	0.039
0.42	0.46	0.40	0.050	0.058
0.59	0.64	0.55	0.077	0.084
0.76	0.80	0.70	0.10	0.11
1.25	1.4	1.0	0.15	0.16
1.8	1.9	1.2	0.17	0.17

The gradation of a given sediment sample is well characterized by the frequency and distribution curves. The standard deviation σ is a rational measure for the width of the frequency curve and increases (for a given average grain size) with increasing gradation. Hence, the ratio σ/d_{50} is suitable for numerical characterization of the gradation of the sand.

Another simple measure of the gradation is the sorting coefficient S defined as the square root of the ratio between two complementary grain sizes, for instance $S = \sqrt{d_{75}/d_{25}}$. River bed material has sorting coeffcients between 1.2 and 3, but most frequently values smaller than 1.6 are found.

It may be convenient to plot the distribution curve on logarithmic probability paper, see figure 2.2.3. If this leads to a straight line, it is concluded that the distribution is log-normal, while

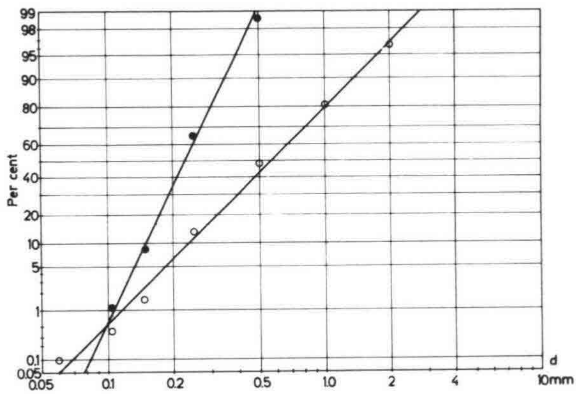

Figure 2.2.3

deviation indicates skewness. The inclination is evidently a measure of the uniformity of the sediment.

2.3. Specific gravity.

The specific gravity γ_s of the grains is the parameter which exhibits the smallest variation under natural conditions.

The ratio

$$s = \gamma_s/\gamma , \tag{2.3.1}$$

in which γ denotes the specific gravity of water at 4°C is called the relative density. In the following we adopt the value $s = 2.65$ for natural sediments. To be in accordance with traditional definitions of dimensionless quantities the symbol s is retained in many of the expressions developed in subsequent sections.

2.4. Settling velocity.

By the settling or fall velocity w of a grain we understand the terminal velocity attained when the grain is settling in an extended fluid under the action of gravity.

The fall velocity w depends on several parameters the most important of which are grain size, specific gravity, shape and the dynamic viscosity of the fluid.

The drag force F on a submerged body is given by the general expression

$$F = c_D \cdot \tfrac{1}{2} \rho \, V^2 \, A ,$$

in which c_D is the drag coefficient, ρ the density of the fluid, V the relative velocity and A the area of the projection of the body upon a plane normal to the flow direction.

Consider now the settling of a single spherical particle of diameter d. The combined action of gravity and boyancy gives the force

$$(\gamma_s - \gamma) \frac{\pi}{6} d^3$$

which under equilibrium conditions must be balanced by the drag, so that we obtain the following equation

$$(\gamma_s - \gamma) \frac{\pi}{6} d^3 = c_D \cdot \tfrac{1}{2} \rho \, w^2 \cdot \frac{\pi}{4} d^2 ,$$

from which

$$w = \sqrt{\frac{4(s-1)\, g\, d}{3 c_D}} \tag{2.4.2}$$

For a single spherical particle in an extended fluid the value of c_D depends on the Reynolds' number

$$\mathbf{R} = \frac{w\, d}{\nu} \tag{2.4.3}$$

exclusively. The relationship is given in double-logarithmic plot in figure 2.4.1.

For very small values of $\mathbf{R}$ the law of Stokes will apply:

$$F = 3\pi \mu \, d \, w , \tag{2.4.4}$$

corresponding to the expression

$$c_D = 24/\mathbf{R} \tag{2.4.5}$$

Under these circumstances eq. (2.4.2) becomes

$$w = \frac{(s-1)\, g\, d^2}{18\, \nu} \tag{2.4.6}$$

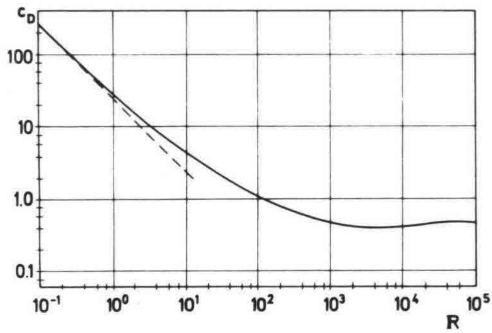

Figure 2.4.1

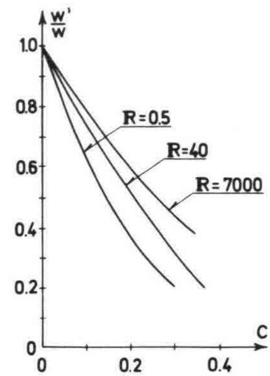

Figure 2.4.2

If the sphere is not single but one of many settling simultaneously, the observed fall velocity w' is smaller than the above expression indicates, the ratio w'/w being a function of the volume concentration c, see figure 2.4.2, which is based on experiments by Richardson and Zaki, [1]. The drag coefficient c_D of a sand particle differs from that of a sphere with the same volume, c_D being dependent on the shape of the grain. In table 2.2.1 are listed a number of fall velocities corresponding to the various sand fractions.

In figure 2.4.3 the value of c_D for a number of typical sand fractions is plotted against the Reynolds' number $\mathbf{R} = w\, d_v/\nu$. For the sake of simplicity c_D is defined by the expression

$$F = c_D \cdot \tfrac{1}{2}\rho\, w^2 \frac{\pi}{4} d_v^2 \tag{2.4.7}$$

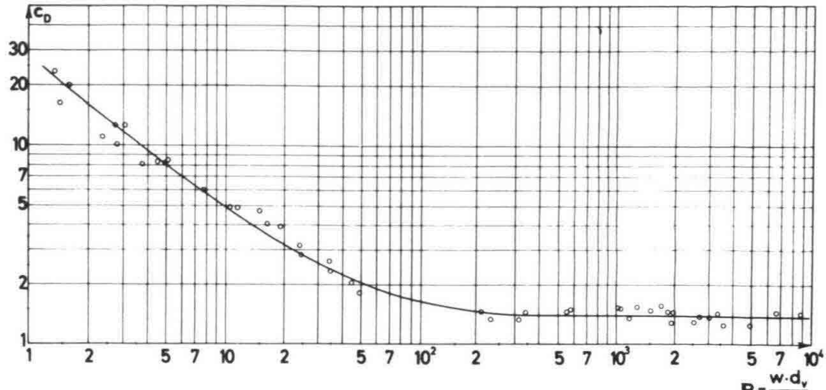

Figure 2.4.3

2.5. Other properties.

A number of sediment properties of secondary importance do not appear explicitely in the theories, either because they are of negligible influence or because our present knowledge of this influence is negligible.

The most important of these properties are possibly the shape and gradation as mentioned above. The neglect of these is probaly among the main reasons for the relatively large scatter experienced, when present sediment transport theories are compared with observations.

Another property of presumingly significant influence is the friction angle (static and dynamic) of the sediment, which in turn depends on the porosity (resp. concentration,).

3. HYDRAULICS OF ALLUVIAL STREAMS.

3.1. Some general definitions.

From the general hydraulic theory we know that in principle it is necessary to make a distinction between hydraulic smooth and rough walls. In open-channel hydraulics, however, there are few exceptions to the rule that the bed must be considered rough, particularly in case of movable bed.

The flow in a prismatic channel is said to be uniform when the mean depth does not change. In alluvial channels the stream bed will usually be covered by dunes, constituting irregularities of a magnitude not negligible as compared with the depth of water, so that - strictly speaking - such flows cannot be uniform in detail. Further, such streams are neither rectilinear nor of constant width. However, in order to develop a simple mathematical description, it is the general practice to neglect minor fluctuations of the cross section and to consider the dunes merely as roughness elements. With such idealizations in mind it becomes possible to maintain the definition of uniformity, and in the following text we only consider flows which may be considered uniform in this sense.

A flow is steady when it does not change in time. In this very rigorous sense hardly any flow occurring in practice is really steady. In alluvial channels the dunes are migrating downstream, changing in size and shape, hence causing a continuous variation of the flow. Fortunately, this change of the flow is always so slow that for all practical purposes it does not affect the applicability of the simple steady-state relationships of elementary hydraulics.

In a steady, uniform open-channel flow the average shear stress τ_0 (tractive force) at the stream bed is found from a simple equilibrium condition to be

$$\tau_0 = \gamma D I \ , \qquad (3.1.1)$$

Hydraulic ?

in which γ is the specific gravity of the water, D is the mean depth and I is the energy gradient (slope). The friction velocity U_f is then defined as

$$U_f = \sqrt{\tau_0/\rho} = \sqrt{g\,D\,I} \tag{3.1.2}$$

The friction factor f is defined by the relation

$$I = f\,\frac{V^2}{2g}\,\frac{1}{D}\ , \tag{3.1.3}$$

where V is the mean velocity and g the acceleration of gravity. From this we find the following simple expression

$$V = \sqrt{\frac{2}{f}\,g\,D\,I} \quad \text{or} \quad V = U_f\sqrt{\frac{2}{f}} \tag{3.1.4}$$

One of the main purposes of the following development is to express the friction factor in terms of the parameters necessary to describe the flow in alluvial channels.

A non-dimensional parameter of basic importance is the Froude number $\mathbf{F}$, defined as

$$\mathbf{F} = \frac{V}{\sqrt{g\,D}} \tag{3.1.5}$$

When $\mathbf{F} < 1$ the flow is said to be subcrtitical, while $\mathbf{F} > 1$ corresponds to supercritical flow. Critical flow occurs when $\mathbf{F} = 1$, so that $V = \sqrt{gD}$. The physical interpretation of this situation is, that the flow velocity V is equal to the celerity

$$c = \sqrt{g\,D}$$

of a surge wave.

When the Froude number is introduced, eq. (3.1.3) is rewritten as

$$I = \tfrac{1}{2}\,f\,\mathbf{F}^2 \tag{3.1.6}$$

The stream bed is called hydraulic smooth if a viscous sublayer is formed. The average thickness δ of the sublayer is usually calculated from the expression

$$\delta = \frac{11.6\,\nu}{U_f}\ , \tag{3.1.7}$$

in which ν is the kinematic viscosity of the fluid. Hence, for increasing velocity δ is decreasing. When δ becomes of a magnitude comparable to that of the roughness elements of the bed the sublayer is "broken". When the quantity calculated by eq. (3.1.7) is smaller than the equivalent sand roughness k of the bed no viscous sublayer will occur, and the bed is said to be hydraulic rough. The transition between smooth and rough conditions depends on the detailed structure and distribution of the roughness elements.

In fully turbulent flow the shear is transferred by the exchange of momentum caused by the turbulent fluctuations. Near a smooth boundary the turbulence ceases and the shear is transferred to the wall through the sublayer essentially by viscous forces.

At a rough wall conditions are different, as no viscous sublayer is present. In this case the shear stress is transferred to the wall as drag (and shear) on the individual roughness elements. The distribution of the velocity U of the mean motion is given by

$$\frac{U}{U_f} = 8.5 + 2.5 \ln \frac{y}{k}$$

in which k denotes the sand roughness and y the distance from the wall, [2]. The average velocity V along the vertical is given by

$$\frac{V}{U_f} = 6 + 2.5 \ln \frac{D}{k} \qquad (3.1.8)$$

The energy dissipation is defined as the part of the total mechanical energy that is transferred into heat. Per unit length area of a uniform channel flow and per unit of time it amounts to $\tau_0 V$. Considering the expression

$$\tau_0 V = \gamma\,(DV)\,I = \gamma\,q\,I$$

we realize that the energy gradient is found as

$$I = \tau_0\, V/\gamma\, q \qquad (3.1.9)$$

3.2. Critical bed shear.

Consider a fluid flow over a bed composed of cohesionless grains. What are the factors determining whether sediment motion will occur or not?

For quite a long time it was common to consider the mean velocity of the flow the determining factor. In this way the concept of a critical velocity was introduced indicating that for velocities greater than the critical sediment motion will occur, while the grains will be immobile for velocities less than the critical. These findings were quite empirical and resulted in critical velocities which varied with the size of the grains as well as the water depth.

Later on a more rational approach to this problem was achieved by considering the dynamic equilibrium of the most exposed bed grains.

Thus let us proceed to discuss the forces acting on the bed grains, see figure 3.2.1

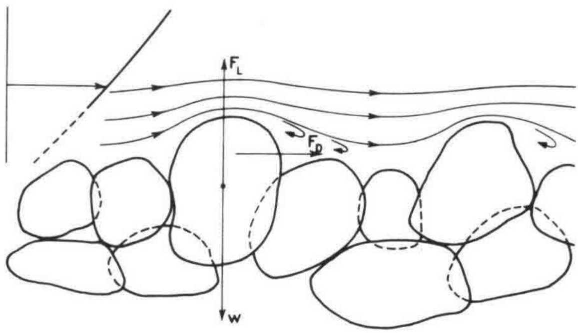

Figure 3.2.1

First we have a horizontal drag F_D created by the flow. F_D consists of a skin friction acting on the surface of the grain and a form drag due to a pressure difference on the up- and downstream sides of the grain. From elementary theory of drag we know that

$$F_D = c_D \tfrac{1}{2} \rho U^2 d^2 \quad ,$$

where U is a characteristic velocity near the bed, d is the grain diameter, and c_D is the drag coefficient which is known to depend on the local Reynolds' number $\mathbf{R} = U d/\nu$ and the grain form. (This dependance is shown for a single spherical particle in figure 2.4.3) In the following it is assumed that the shape effects are accounted for sufficiently well by using the fall diameter.

Taking the friction velocity $U_F = \sqrt{\tau_0/\rho}$ as the characteristic velocity U we find

$$F_D \sim c_D(\mathbf{R}) \, \tau_0 \, d^2 \qquad (3.2.1)$$

Thus F_D is found to be proportional to the bed shear $\tau_0 = \gamma\, D\, I$.

Generally also a lifting force F_L in excess of the natural boyancy is created by the flow. This lift is primarily due to deviations from the hydrostatic pressure distribution around the grain. Near the upper part of the grain the pressure will locally decrease below the hydrostatic pressure, while an excess pressure will be created at the lower part of the grain.

In order to understand the appearance of such pressure deviations let us consider a simple model consisting of uniform spheres arranged in a hexagonal pattern on a plane, see figure 3.2.2.

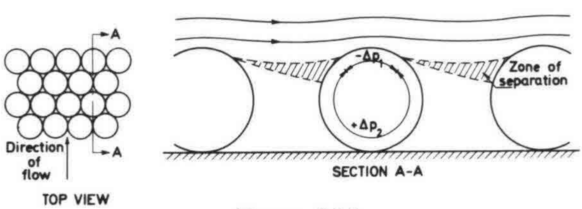

Figure 3.2.2

Near the bed the flow will have a curvilinear character, i.e. just above a sphere it will be upward convex, while it will be downward convex between two grains, see figure 3.2.2. The fluid in between the spheres is assumed to be motionless and separated from the flowing fluid by a zone of separation.

In order to make such a curvilinear flow possible we must have some balancing centripetal forces proportional to $\Delta p_1\, d^2$ and $\Delta p_2\, d^2$, respectively, where Δp_1 is the underpressure at the top of a sphere and Δp_2 the excess pressure in the fluid at rest between the spheres. Taking $U_F = \sqrt{\tau_0/\rho}$ as a characteristic velocity near the bed it is easily found from dimensional reasons that the corresponding centrifugal forces are proportional to

$$\rho\, d^3\, (U_F^2/d) = \rho\, U_F^2\, d^2 \quad .$$

Thus we have

$$\Delta p_1 \sim \Delta p_2 \sim \tau_0$$

from which we obtain for F_L

$$F_L \sim (\Delta p_1 + \Delta p_2) d^2 \sim \tau_0 d^2 \tag{3.2.2}$$

that is proportional to the bed shear stress τ_0.

In case of a natural grain bed the situation is much more complicated. However, qualitatively eq. (3.2.2) may still be considered valid so that it can be concluded that the agitating forces F_D and F_L both vary in proportion to the shear τ_0.

The immersed weight of a grain $W \sim (\gamma_s - \gamma)\, d^3$ is the stabilizing force on a grain.

Now it is evident that the mobility of the grains depends upon the relative size of the forces F_D, F_L and W. Thus it is natural to introduce a parameter θ representing the ratio between the agitating forces (F_D and F_L) and the stabilizing force (W). We define

$$\theta = \frac{\tau_0 d^2}{(\gamma_s - \gamma) d^3} = \frac{\tau_0}{\gamma(s-1)\, d} = \frac{D\, I}{(s-1)\, d} = \frac{U_f^2}{(s-1) g\, d} \tag{3.2.3}$$

θ is seen to be a dimensionless form of the bed shear τ_0. As mentioned above the grain begins to move if the ratio θ exceeds a certain critical value θ_c, which may be some function of **R**, so that

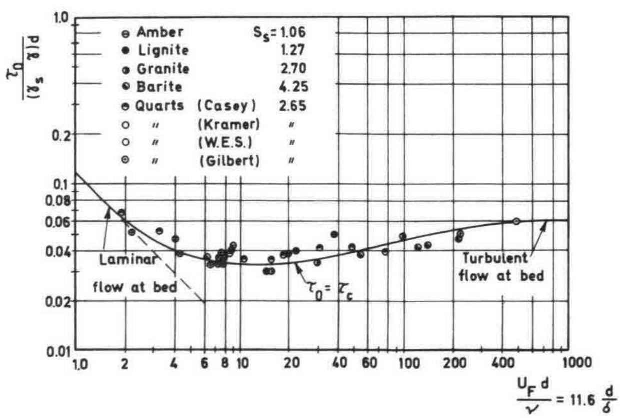

Figure 3.2.3

$$\theta_c = f_1 \left(\frac{U_F d}{\nu}\right) = f_2 \left(\frac{d}{\delta}\right) \tag{3.2.4}$$

where the thickness δ of the viscous sublayer has been introduced by substitution of eq. (3.1.7). The relation (3.2.4) was first pointed out by Shields, [3]. An experimental determination was also made by Shields in case of uniform grains. The result is shown in figure 3.2.3. The bed shear τ_c corresponding to incipient grain motion is denoted the critical bed shear. From eq. (3.2.3) we obtain

$$\tau_c = \theta_c \, (\gamma_s - \gamma) \, d \tag{3.2.5}$$

Shields' experiments showed that for large values of $U_F \, d/\nu$ we have $\theta_c \simeq 0.06$.

3.3. Transport mechanisms.

Of the total sediment load a distinction in two categories is made, the bed load and the suspended load. No precise definitions of these terms have been given so far, but the basic idea in splitting up the total sediment load in two parts is that roughly speaking two different mechanisms are effective by the transport.

The bed load is defined as the part of the total load that is in more or less continuous contact with the bed during the transport. It primarily includes grains that roll, slide or jump along the bed. Thus the bed load must be determined almost exclusively by the effective bed shear acting direct on the sand surface.

The suspended load is the part of the total load that is moving without continuous contact with the bed, as a result of the agitation of fluid turbulence.

For given hydraulic and sediment characteristics what is the rate of sediment transport in an alluvial channel?

This question has been challenging a great number of scientists for nearly a century.

The first important analysis of the problem of bed load transport is due to Du Boys (1879), [4]. His approach is based on the over-simplified picture of mutually sliding layers of bed material.

H. A. Einstein departed radically from the Du Bous type of analysis in his approach to the problem of bed load transport, [5]. Thus he introduced a theory of probability in order to account for the statistical variation of the agitating forces on bed grains caused by the turbulent fluid motion.

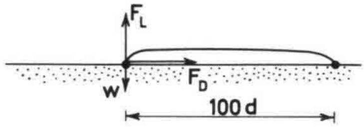

Figure 3.3.1

If the magnitude of the instantaneous agitating forces on a certain bed particle exceeds that of the corresponding stabilizing forces the particle is eroded and begins to jump, roll or slide along the bed until it finally is deposited at a locality where the magnitude of the instantaneous agitating forces is smaller than that of the stabilizing forces. On experimental basis Einstein assumes that the mean distance travelled by a sand grain between erosion and subsequent deposition is proportional to the grain diameter and independent of the hydraulic conditions and the amount of sediment in motion. Einstein suggests a constant of proportionality of approximately 100.

The statistical point of view throws a new light on the concept of critical bed shear. A critical mean bed shear for the whole channel is no longer relevant as there will always be a certain positive probability for the instantaneous shear stress at some locality to exceed any finite value and thus cause particle motion. At a given locality of the bed, however, it still gives a meaning to talk about a critical shear stress, which becomes surpassed every time a particle is eroded from this locality.

The principle in Einstein's procedure is the following: He considers what happens within a unit area of the bed. The number of grains deposited within this unit area depends on the number of grains in motion (that is of the rate of transport) as well as on the probability that the dynamical forces admit the grains to deposit. The number of grains eroded from the same unit area depends on the number of particles within this area as well as on the probability that the hydrodynamic forces on these grains are sufficiently strong to move them. For equilibrium conditions the number of grains deposited must equal the number of grains eroded.

In this way a functional relationship is obtained between the two dimensionless quantities

$$\Phi_B = \frac{q_B}{\sqrt{g(s-1)\,d^3}} \qquad \text{and} \qquad \theta = \frac{\tau_0}{\gamma\,(s-1)\,d} = \frac{D\,I}{(s-1)\,d}$$

q_B is the rate of bed load transport in volume of material per unit time width of section. Thus Φ_B is a dimensionless form of the bed load discharge, while θ is the previously introduced dimensionless form of the bed shear, eq. (3. 2. 3).

Einstein's Φ_B-θ relationship is a rather complicated expression . However, for small sediment discharges $\Phi_B < 10$, it follows rather closely the simple relation

$$\Phi_B = 8(\theta - 0.047)^{3/2} \tag{3. 3. 1}$$

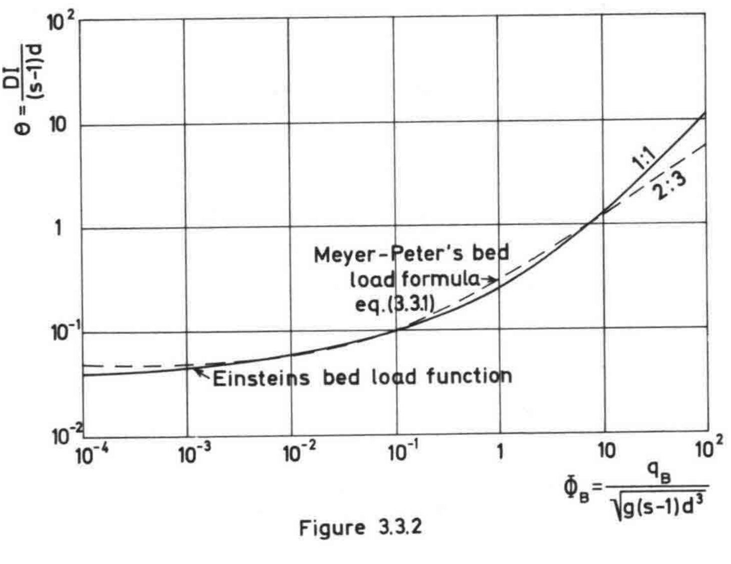

Figure 3.3.2

This expression is obtained by Meyer-Peter [6] on the basis of extensive flume experiments.

Thus for small bed load discharges Einstein's theory predicts the transport rate very well. For large amounts of bed load ($\Phi_B > 10$), however, a significant deviation between Einstein's bed load function and experiments is found.

Let us consider the ideal case of a fluid flow over a bed of uniform spheres, perfectly piled and therefore equally exposed. All statistical variations due to turbulence are disregarded.

What happens if the bed shear exceeds the corresponding critical value? As the particles in the upper layer are equally exposed the whole top layer is then peeled off simultaneously and becomes dispersed. Hence the next layer of particles is exposed to the flow and should consequently also be peeled off. Thus the immediate consequence is that all the subsequent underlying layers of grains are also eroded, so that no grain bed could exist at all, when the bed shear exceeds the critical. However, general experience tells us that a grain bed exists for all practical situations.

This paradox may have been one of the main reasons for the basic works of R. A. Bagnold [7] and [8]. In the first of these works Bagnold considers the dynamics of a fluid-particle mixture in shearing motion under several ideal conditions. In the second work he applies the results obtained in [7] to the specific case of a water-sediment mixture flowing over a gravity bed.

Bagnold introduces several assumptions, the following two being the most fundamental:

(i) The total overall shear stress τ in the mixture is separable in two parts, viz.

$$\tau = \tau_F + \tau_G \tag{3.3.2}$$

where τ_F is the shear stress transmitted through the intergranular fluid. Correspondingly τ_G is the shear stress transmitted because o the interchange of momentum caused by the encounters of the grains (tangential dispersive stress).

(ii) Also a normal stress (dispersive pressure) σ_G is developed in a shearing fluid-particle mixture because of the encounters of the grains. τ_G and σ_G are linked together by a relationship of the form

$$\tau_G = \sigma_G \tan \alpha \qquad (3.3.3)$$

where tan α is a dynamic analogue of the static friction coefficient. From experiments Bagnold finds tan α to take on values between 0.32 and 0.75 corresponding to encounter conditions where inertial or viscous effects are dominating, respectively.

In relation to assumption (i) the separation of the total shear stress is sketched for a uniform channel flow for which the total

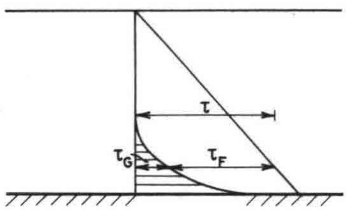

Figure 3.3.3

shear stress is known to vary linearly. The actual distribution of τ_G naturally depends on the amount of sediment in motion.

According to Bagnold the immersed weight of the suspended load is balanced exclusively by an excess static fluid pressure, while the immersed weight of the bed load is transmitted to the stationary grains of the fixed bed by the dispersive pressure induced by the collisions of the dispersed grains.

Within the framework of Bagnold's theory as outlined above it is now easy to explain the paradox described above.

When a layer of spheres is peeled off some of the spheres will go in suspension, while others will be transported as bed load. Thus a dispersive pressure on the next layer of grains will develop. This means, however, according to eqs. (3.3.2) and (3.3.3) that a certain part of the total bed shear is transmitted as a grain shear stress τ_G and a corresponding minor part as the fluid stress ($\tau_F = \tau - \tau_G$). Continuing this argumentation it will be understood that exactly as many layers of grains will be eroded that the residual fluid stress $\tau_F = \tau - \tau_G$ on the first immovable layer equals the critical bed shear stress τ_c.

The mechanism in the transmission of a fluid shear stress τ greater than the critical to the fixed bed is then the following: The part τ_c is transferred direct from the fluid to the immobile bed. The residual stress $\tau - \tau_c$ is transferred (by drag) to the moving particles and further from these to the fixed bed as a tangential dispersive stress.

Finally in this section we will deduce the classical differential equation for the distribution of suspended load in a steady, uniform and two-dimensional open channel flow.

Let c denote the temporal mean value of the concentration of suspended load at a certain height y above the bed. We consider a unit area parallel to the bed at the same height y.

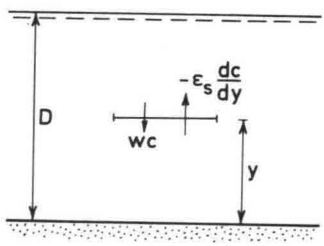

Figure 3.3.4

Because of the settling of the suspended grains this area is passed in the downward direction per unit of time by the following amount ($m^3/s \cdot m^2$) of grain material

$$w\,c$$

where w is the settling velocity of the grains. Because of the turbulent motion a (turbulent)diffusion of suspended load will occur in the direction of decreasing values of c, i.e. upwards. This diffusion is to a first approximation proportional to the rate of change of c. Thus we find that the amount of sediment lifted through the unit area equals

$$- \epsilon_S \frac{dc}{dy}$$

where ϵ_S is the diffusion coefficient for suspended load. The negative sign is due to the fact that dc/dy is negative. in the direction of the diffusion transport

Thus thecondition for an equilibrium distribution of suspended load is given by the following differential equation

$$\epsilon_S \frac{dc}{dy} + w\,c = 0 \tag{3.3.4}$$

ϵ_S is an unknown quantity for which we have to make assumptions before eq. (3. 3. 4) can be solved.

Usually it is assumed that ϵ_S is equal to (or proportional to) the eddy viscosity for the flow, so that

$$\epsilon_S \sim \epsilon = \kappa\, U_f\, y(1 - \frac{y}{D}) \quad ,$$

in which κ is v. Kármán's universal constant, approximately equal to 0. 4. Substitution into eq. (3. 3. 4) and separation gives

$$\frac{dc}{c} = - \frac{w}{\kappa\, U_f} \frac{dy}{y(1 - \frac{y}{D})}$$

or by integration

$$c = \text{const.} \cdot \left(\frac{D-y}{y}\right)^{z} \quad , \text{ where } z = \frac{w}{\kappa U_f} \tag{3.3.5}$$

This expression is due to H. Rouse [9] and has been confirmed experimentally by V. Vanoni, [10].

No general method for prediction of the constant in eq. (3. 3. 5) seems to be available to-day.

3.4. Bed configurations.

A plane stream bed will usually not be stable, but tends to break up and form one of the bed configurations discussed below. To give a general review of the most important possibilities we imagine the experiments in a flume in which the velocity is gradually increased, see figure 3. 4. 1. If the bed shear stress is plotted against velocity we get a reasonably well-defined relationship suitable for a general review. The dotted curve indicates the pure skin friction τ', see section 3. 6.

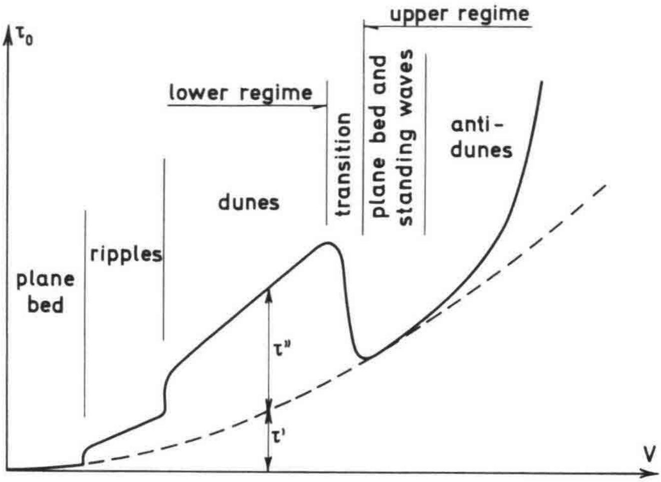

Figure 3.4.1

Plane bed.

When the velocity is so small that the bed shear stress is below the critical value no sediment transport is taking place and the bed will consequently remain plane.

Ripples.

When the critical tractive force is exceeded so that sediment transport starts, the bed will be unstable. In case of fine sediment, ripples are formed, while coarse sediments usually will form dunes.

Ripples is the notation of small triangular-shaped sand waves, usually shorter than about 0.6 meters and not higher than about 60 mm, see figure 3.4.2.

As mentioned in section 3.1 a viscous sublayer is formed when the flow velocity is small, while it gradually disappears for increasing velocity. It is natural ? to assume that ripples are formed if a viscous layer is present when the critical tractive force is just surpassed, while dunes are formed if the bed is hydraulic rough. An indication in this direction is obtained from table 3.4.1, in which the limit between ripples and dunes has been found from flume experiments [11] and the corresponding values of **R** (= Reynolds' number for the grains)is indicated.

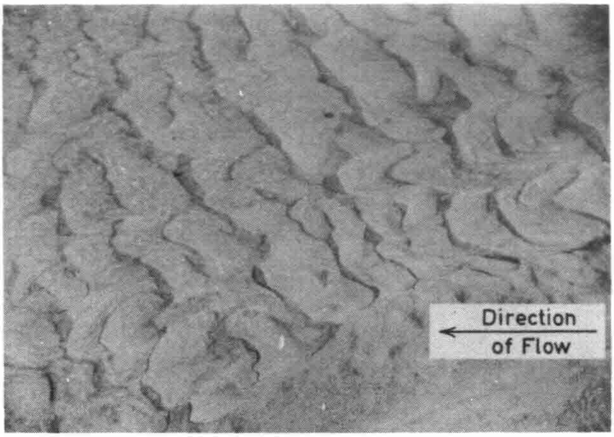

Figure 3.4.2

Table 3.4.1

Value of **R** = $U_f d/\nu$ for limit between ripples and dunes

d mm	0.19	0.27	0.28	0.45	0.47	0.93
R	7.3	10.3	11.0	11.7	12.2	-

For the coarsest sand no ripples are formed. Although the critical value of **R** is not completely constant, this table gives a first crude criterion for the transition between ripple and dune formation. Note, that a critical value of **R** = 11.6 corresponds to d = δ, cf eq. (3.1.7)

Dunes.

The large, more or less irregular, sand waves usually formed in natural streams are called dunes, see figure 3.4.3.

The longitudinal profile of a dune is often nearly triangular, having a slightly curved upstream surface and a downstream slope approximately equal to the friction angle (angle of repose) of the bed material figure 3.4.4.

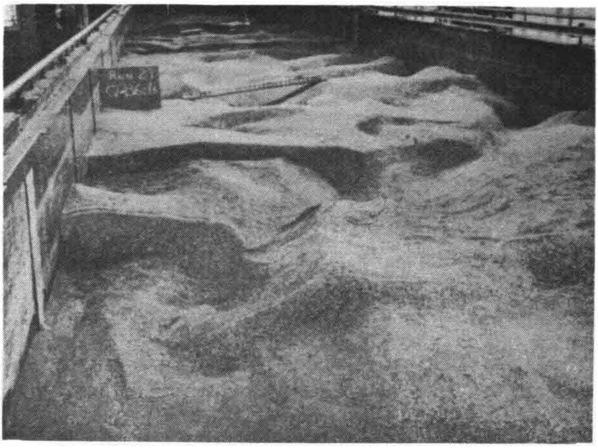

Figure 3.4.3
(By permission reproduced from [31])

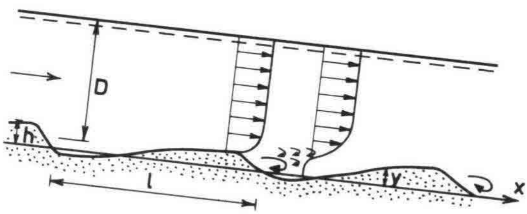

Figure 3.4.4

At the lee side of each dune a bottom roller is formed. Above this - in the immediate continuation of the crest - a zone of violent free turbulence is formed, in which the production of turbulent energy is very large. The conditions are very similar to those found in other well-known cases of free turbulent flow as for instance in jets and wakes. Near the trough the sediment particles are moved by this turbulence, probably even when the local shear stress is below the critical, [12]. On the main part of the dune the shear stress is moving the particles along the inclined surface until they roll over the crest and become buried in the bed for a while. As particles are eroded from the stream side and deposited at the lee side of the dune, we get a continuous downstream migration of the dunes.

To investigate this transport a little closer let us do the simplifying assumption that the dunes are moving at a constant velocity a and without any change in shape. This is of course not true in detail but represents all the essential features of the problem with sufficient accuracy. Under these conditions the shape of the bed is described by an expression of the form

$$y = \varphi(x - at) \tag{3.4.1}$$

in which t is the time, y the local height of the dune above an x-axis placed through the troughs, see figure 3.4.4. Next we consider the sediment discharge q_T at two consecutive sections, spaced a unit length in the x-direction.

The net outflow of sediment is

$$\frac{\partial q_T}{\partial x},$$

which must equal the change in bed elevation when correction for the porosity n of the bed material is considered. Hence, we get the equation of continuity:

$$\frac{\partial q_T}{\partial x} = -(1 - n)\frac{\partial y}{\partial t} \tag{3.4.2}$$

If eq. (3.4.1) is substituted into eq. (3.4.2) it becomes obvious that the equations are satisfied if we put

$$q_T = q_T(x) = q_0 + a(1 - n)\,y \tag{3.4.3}$$

The quantity q_0 is a constant, immediately interpreted as the value of q_T for $y = 0$, i.e. at the troughs where the bed load vanishes. q_0 is the part of the total sediment discharge that does not take part in the bed process, and it is consequently identified with the suspended load.

For the bed load we then obtain the interesting relation

$$q_B = a(1 - n)\,y \tag{3.4.4}$$

from which we see that the local intensity of bed load transport is proportional to the local height of the bed above the plane through the troughs.

This indicates that the shear stress τ' at the dune surface must vary from zero at the trough to a maximum at the crest, an assumption confirmed by measurements published by A. J. Raudkivi, [12].

From the above description it appears that the hydraulic resistance is determined by two types of roughness elements of very different scale, the grains and the dunes.

A transport process of this type will cause local segregation of the bed material if the gradation is large, so that the material found in the crest is finer than the material found at the trough.

The dune pattern is often irregular and continuously changing, so that dune height and length are stochastic variables more or less difficult to define exactly. A rational approach has been presented by C. F. Nordin and J. H. Algert [13], who suggested a spectral analysis of the pattern. It often happens that smaller dunes are formed at the streamside of larger ones, moving onto the crest and changing the height and migration velocity of these.

At the smaller transport ratio ripples may occur locally, particularly near the troughs.

Transition and plane bed.

Through a region of transition a state of plane bed is approached. This state is really stable in the sense that small disturbances will be smoothed out rapidly. For fine sediments and large depths the transition zone is very narrow, while it may be very extended and diffuse in case of coarse material and small depths.

The question of plane bed contra the formation of sand waves has been treated as a problem of stability, [14], a semi-empirical approach yielding a rough account of the most important features of the bed configurations. The basic idea of the stability analysis is that the originally plane sand bed is assumed to be deformed into a two-dimensional sinusoidal shape. If the amplitude of this perturbation - by the application of the flow equations - is found to increase in time, the plane bed is evidently unstable, while a decrease indicates stability. For the so-called neutral disturbances the amplitude remains unchanged corresponding to the transition between stable and unstable conditions.

Linear stability theory can only answer the question of initial stability that is stability for infinitely small disturbances. When the amplitude becomes finite secondary effects intervene, so that the further development of the perturbation is determined by the balance between the originally agitating mechanism and the the damping originating from the secondary effects not accounted for in the linear theory.

If the plane bed is actually unstable, two different possibilities are open. In one case separation will occur at the downstream side of the bed ondulations, and the formation of dunes (or ripples) will be the result. In the other case the secondary effects will stop the exponential amplification of the amplitude, so that a finite amplitude is reached (standing waves).

A similar analysis is given by J. F. Kennedy, [15] and O. Reynolds, [16].

The transition zone is said to separate the lower and the upper flow regime, see figure 3.4.1.

Anti-dunes.

For increasing flow velocity an unsteady flow pattern corresponding to the so-called anti-dunes is formed. The bed profile and the water surface are both roughly sinusoidal, but out of phase.

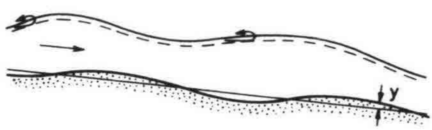

Figure 3.4.5

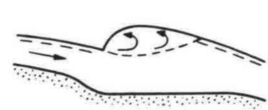
Figure 3.4.6

At higher Froude numbers the amplitude of the surface profiles tend to grow until breaking occurs, as indicated in figure 3.4.5. After breaking the amplitude may be small for a while, and then the process of increase and breaking is repeated. The name anti-dune indicates the fact that the bed and surface profiles are moving upstream, particularly immediately before breaking.

Previously we found that the rate of sediment transport q_T was given by eq. (3.4.3):

$$q_T = q_0 + a(1 - n)\, y.$$

As a is negative for anti-dunes we conclude that q_T is minimum at the crests and maximum in the troughs.

An extreme form of the anti-dune, occurring at high velocities (large Froude numbers) is the chute- and -pool flow indicated in figure 3.4.6. In this flow series of steep chutes with supercritical flow are separated by pools in which hydraulic jumps are formed.

Prediction of bed configuration.

The short account of the various bed forms given above does not make it possible to predict the precise flow conditions for a given channel flow. Although a diagram as that in figure 3.4.1 is often rather well defined, it should be born in mind that it is different for different sediment sizes. However, by the introduction of suitable dimensionless variables, these different diagrams may be mapped into a single plot common for all sediment sizes, except as regards the transition zone, see section 4.

From figure 3.4.1 it is evident (as pointed out by N. H. Brooks, [17]), that the flow velocity is no single-valued function of the bed shear. However, for a given flume (or channel) and bed material the flow is uniquely determined from the discharge Q and the rate of total sediment transport Q_T, wash load being neglected. This is utilized in the design chart presented in figure 6.2.1, where also suitable non-dimensional variables are applied.

3.5. Shape effects.

The shape of the cross section has a certain, but generally not dominating effect on the bed configuration and sediment transport rate.

At first we consider the flow in rectangular flumes. If the width of the flume is infinite, we have a situation like that in figure 3.5.1, where the shear is uniform equal to γDI. For a flume of finite width

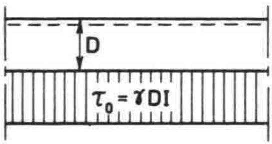

Figure 3.5.1

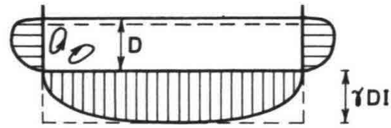

Figure 3.5.2

some of the shear stress is necessarily transferred to the side walls, so that the bottom shear will be less than γDI, at least near the walls, and in narrow flumes all over the bed (figure 3. 5. 2).

Firstly this reduces the average bed shear, secondly it makes the distribution of the shear stress non-uniform. The reduction depends on the depth-width ratio of the flume and on the hydraulic roughness of bed and side walls. Because of this shape effect the results from narrow experimental flumes should be considered with some reservation, even when theoretical side-wall correction has been performed.

Another phenomenon caused by the existence of side walls is the secondary currents, first described by L. Prandtl, [18]. The effect on the transportation of suspended load has been discussed by Chiu and McSparrow, [19].

Most natural streams are shallow in the sense that the mean depth D is small as compared with the surface width B. If the local depth is y, see figure 3. 5. 3, the bed shear stress is found to be

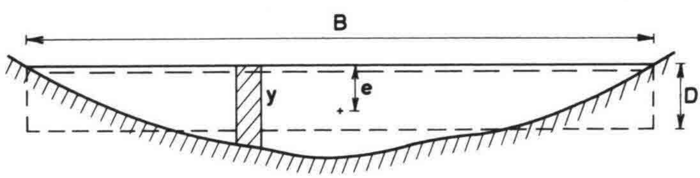

Figure 3.5.3

$$\tau_0 = \gamma\, y\, I \ , \tag{3.5.1}$$

if the shear forces in vertical sections are neglected. By integration all over the wetted parameter P we get

$$\int_P \tau_0 \, d\, P \simeq \tau_m \, P \simeq \tau_m \, B = \gamma\, I\, A \ ,$$

so that the mean shear stress τ_m becomes

$$\tau_m = \gamma\, I\, A/B = \gamma\, D\, I \tag{3.5.2}$$

From this simple analysis it is evident that the shape is a factor of some importance. The shear stress varies in linear proportion to the depth according to eq. (3.5.1), which means that the bed configuration may change considerably in a given cross section. For instance, it may happen that plane bed and standing waves occur in the middle part of the streams, while dunes or ripples are formed near the banks.

The flow in natural streams is often referred to flow in rectangular flumes in the simple way that variations are neglected, so that the mean depth and shear stress are applied, when hydraulic resistance and sediment transport are calculated. Even when the bed is covered by dunes in the whole cross section, this method is rather crude. The authors suggested on the basis of a theoretical analysis [14], that D should be substituted by the quantity 2e, where e is the depth below water surface of the "centre of gravity" of the cross section.

For a rectangular cross section we get 2e = D, while for a shallow triangular we get 2e = 1.33 D. Hence, in general the shape effect caused by the non-uniform distribution of the boundary shear stress is not negligible. However, no experimental investigations seem to exist and the lack of exact knowledge of the hydraulic resistance of alluvial streams makes it acceptable at the present state of development to neglect the shape effect.

Secondary currents occur in natural streams as well as in flumes, particularly in relation to bends. The secondary currents induced by meandering of the streams have a pronounced influence on the shape of the cross section, as indicated in figure 3.5.4.

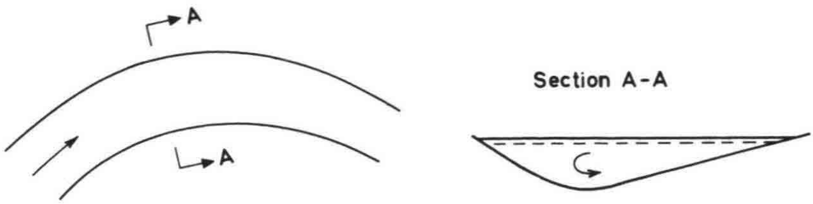

Figure 3.5.4

3.6. The effective bed shear stress τ'.

Dunes are the bed configuration that is by far the most important in practice. Unfortunately it is also the most difficult case to deal with theoretically, because the dune pattern is not known beforehand and because it changes in a complicated manner when flow conditions are changed.

It was mentioned in section 3.4 that the hydraulic resistance originates from two different types of roughness elements of very different scale. The small scale roughness is the sediment grains at the surface of the dunes, while a dune in itself acts as macroscopic roughness element.

In accordance with this picture the total bed shear stress $\tau_0 = \gamma D I$ is divided in two different parts

$$\tau_0 = \tau' + \tau'' , \qquad (3.6.1)$$

of which τ' is the mean value of the part acting directly as a friction on the surface of a dune, while the residual part τ'' corresponds to a "form drag" on the dunes, due to the fact that the water pressure is larger at the rear side than at the lee side. It is evident that the transport of bed load must be related to τ' only, for which reason this is called the effective bed shear.

From the definition of the friction factor f we get

$$\tau_0 = f \tfrac{1}{2} \rho V^2 , \qquad (3.6.2)$$

and similarly

$$\tau' = f' \tfrac{1}{2} \rho V^2 \qquad (3.6.3)$$

A more profound understanding of the flow over a dune-covered bed is obtained by the discussion of figure 3.6.1, which is a vertical longitudinal section through the channel.

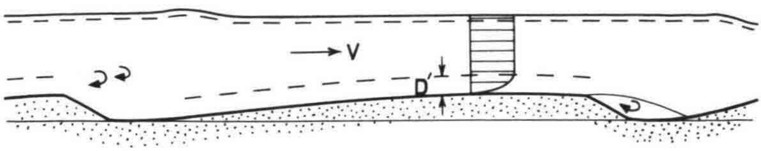

Figure 3.6.1

Immediately downstream the crest a wake-like flow is formed, in which a large amount of turbulent energy is produced. This is dissipated into heat further downstream, thus causing the well-known "expansion loss". At high Froude numbers a boil is formed at the water surface a little downstream from the expansion.

At the end of the trough a boundary layer is formed, in which the velocity gradient is large, while the velocity distribution outside this layer is very uniform.

Now it is natural to assume that the upper flow and the boundary layer flow are independent of each other in the sense that no essential amount of energy is exchanged between them. Hence, the energy gradient of the boundary layer flow (defined as the dissipation divided by unit weight and discharge, see eq. (3. 1. 9)) must equal that of the upper layer and that of the total flow.

$$I = \frac{\tau' \ V'}{\gamma V' \ D'} = \frac{f' \ \frac{1}{2} \rho \ V^2}{\gamma \ D'} = \frac{f' \ V^2}{2g \ D'} \ ; \qquad (3.6.4)$$

in which V' is the mean velocity in the boundary layer. As also

$$I = \frac{f \ V^2}{2g \ D} \ ,$$

we get the expression

$$\frac{f'}{D'} = \frac{f}{D} \qquad (3.6.5)$$

From this and the equation

$$\sqrt{\frac{2}{f}} = \frac{V}{\sqrt{g \ D \ I}}$$

we further obtain

$$\sqrt{\frac{2}{f'}} = \frac{V}{\sqrt{g\,D'\,I}}$$

The friction factor f' for the boundary is determined by a formula of the type

$$\sqrt{\frac{2}{f'}} = c + 2.5 \ln \frac{D'}{k} \ ,$$

in which k is the equivalent sand roughness as defined by Nikuradse, [2], while c is a constant, depending on the unknown velocity distribution in the layer. As we must have $f = f'$ for $D = D'$ we take $c = 6$ and get finally

$$\frac{V}{\sqrt{g\,D'\,I}} = 6 + 2.5 \ln \frac{D'}{k} \ . \tag{3.6.6}$$

This equation was originally suggested by H. A. Einstein [5], who obtained it as an analogy to his method of calculating side wall correction. The present method, however crude, has the immediate advantage of giving an interpretation of D' as the boundary layer thickness. As to experimental support reference is made to a paper by Meyer-Peter and Müller, [6], who developed an expression of different appearance, but numerically very close to that of Einstein.

Combining eqs. (3.6.2), (3.6.3) and (3.6.5) we get the important expression

$$\tau' = \gamma\, D'\, I \ . \tag{3.6.7}$$

Finally the magnitude of the sand roughness k should be discussed. For a plane non-moving sand bed it follows from Nikuradse's experiments, that we should expect $k \simeq d$. In case of moving sand grains we can get some information of the roughness by investigation of flows without expansion losses, that means flows with plane bed and standing waves. An analysis has indicated a value of $k \simeq 2.5\, d$ on an average, where d denotes the mean fall diameter.

For later use we note that the logarithmic expression (3.6.6) may be approximated by a power formula for a rather wide interval of D'/k. For most practical cases we may put

$$\frac{V}{\sqrt{g D' I}} = 9.45 \left(\frac{D'}{k}\right)^{1/8} ,$$

approximating eq. (3. 6. 6) within a 5 pct. margin of error in the interval

$$13 < D'/k < 1.5 \cdot 10^4$$

This gives the simple formula

$$V = \frac{9.45\sqrt{g}}{\sqrt[8]{k}} D'^{5/8} I^{1/2} \tag{3. 6. 8}$$

4. THE SIMILARITY PRINCIPLE.

4.1. Basic parameters.

When confronted with the problem of collecting extensive empirical data into a consistent system the first aim must be to obtain a system of non-dimensional parameters just sufficient to characterize the flow system completely. The approach presented below makes use of simple similarity considerations, which is feasible when the effect of viscous shear is negligible, i. e. for dunes and the upper flow regime . In case of ripples a more or less pronounced scale effect must be expected.

At first, however, it is convenient to derive a simple equation for the hydraulic resistance. In case of a dune-covered bed it was mentioned in section 3. 6, that one part of the total loss of mechanical energy is due to the flow expansions after each crest and another part to friction. In case of anti-dunes the flow does not separate from the bed, but breaking at the water surface causes a loss similar to the expansion loss. In the following the arguments concentrate on the dune case, but some of the final results are sufficiently general to account for essential features of the upper flow regime too.

The magnitude of the expansion loss $\Delta H''$ may be estimated from Carnot's formula

$$\Delta H'' = \alpha \frac{(U_1 - U_2)^2}{2g} \tag{4. 1. 1}$$

in which U_1 is the velocity before the expansion (i. e. above the crest) and U_2 the smaller mean velocity downstream from the crest. α is a non-dimensional coefficient depending on the flow geometry.

When the dune height is denoted h and the mean depth D, we get

$$U_1 = \frac{q}{D - \frac{1}{2}h} \quad \text{and} \quad U_2 = \frac{q}{D + \frac{1}{2}h} ,$$

so that eq. (4. 1. 1) becomes

$$\Delta H'' = \frac{\alpha q^2}{2g} \left[\frac{1}{D - \frac{1}{2}h} - \frac{1}{D + \frac{1}{2}h}\right]^2 \simeq \frac{\alpha V^2}{2g} \left(\frac{h}{D}\right)^2 \qquad (4. 1. 2)$$

The energy gradient I is the energy loss per unit length of the channel. Hence, we get

$$I = I' + \frac{\Delta H''}{l} , \qquad (4. 1. 3)$$

in which I' is the gradient due to friction and l is the dune length. This expression is now compared with eq. (3. 6. 1):

$$\tau_0 = \gamma D I = \tau' + \tau''$$

or

$$I = \frac{\tau'}{\gamma D} + \frac{\tau''}{\gamma D} ,$$

in which the last term may be identified with the last term in eq. (4. 1. 3) as both pertain to the expansion loss. Consequently, we find

$$I = \frac{\tau'}{\gamma D} + \frac{1}{2} \frac{V^2}{gD} \frac{\alpha h^2}{Dl} \qquad (4. 1. 4)$$

Recalling eq. (3. 6. 7) we see that,

$$\tau' = \gamma D I' = \gamma D' I .$$

An expression for the friction factor f is obtained, if eq. (4. 1. 4) is divided by $\frac{1}{2}\mathbf{F}^2$:

$$f = f' + \frac{\alpha h^2}{D l} \qquad (4. 1. 5)$$

From this or from eq. (4. 1. 4) we may obtain another important relationship between dimensionless parameters

$$\theta = \theta' + \theta'' ,$$

in which

$$\theta = \frac{D I}{(s-1)d} , \quad \theta' = \frac{D' I}{(s-1)d} \quad \text{and} \quad \theta'' = \frac{1}{2}\mathbf{F}^2 \frac{\alpha h^2}{(s-1)d\, l} .$$

Next we consider two different flow systems, 1 and 2, that are supposed to be geometrical similar, with the scale ratio $\lambda = D_1/D_2$, see figure 4. 1. 1.

Figure 4.1.1

Then we proceed to investigate the conditions for complete similarity, assuming the bed configuration to be dunes. Geometric and dynamic similarity demands:

1) $\frac{D_1}{d_1} = \frac{D_2}{d_2}$ or $\lambda = \frac{D_1}{D_2} = \frac{d_1}{d_2}$

2) Equal slopes: $I_1 = I_2$

3) The dimensionless effective bed shear must be the same: $\theta'_1 = \theta'_2$.

When these conditions are fulfilled the dune patterns will be similar. It follows from 1) and 2) that $\theta_1 = \theta_2$, so that also $\theta''_1 = \theta''_2$. From this follows that, the expansion loss is the same fraction of the total loss of mechanical energy in both streams.

The variation of the relative density s is neglected.

To be rigorous the grain size distribution should be assumed similar also. However, practical evidence indicates the gradation to be of secondary importance, so that we as a first approximation may characterize the sediment by the mean fall diameter d, exclusively.

For two similar streams we find from eq. (4. 1. 5) and eq. (3. 6. 6) that the friction factor is the same. As also the slopes are common we conclude that the flows take place in such a way that the Froude numbers are identical.

From this discussion we must assume that for a given sediment size d the flow system is completely defined if the quantities D, I and θ' are specified.

Now we are able to calculate the scale ratio of sediment transport rates:

$$\frac{q_{T1}}{q_{T2}} = \frac{U_{f1}\, d_1}{U_{f2}\, d_2}$$

From eq. (3. 2. 3) we get $U_f = \sqrt{\theta}\sqrt{(s-1)g\, d}$, so that

$$\frac{q_{T1}}{q_{T2}} = \sqrt{\frac{(s_1-1)g_1\, d_1{}^3}{(s_2-1)g_2\, d_2{}^3}}$$

From this we obtain

$$\Phi_1 = \Phi_2 \quad , \quad \text{as} \quad \Phi = \frac{q_T}{\sqrt{(s-1)g\, d^3}}$$

It follows that the non-dimensional transport rate Φ must be a function of the determining parameters:

$$\Phi = \Phi\,(\frac{D}{d}\ , I, \theta')\quad \text{or} \quad \Phi = \Phi\,(\theta, \frac{D}{d}, \theta')$$

In the next section we find that θ' is actually a function of θ, so that

$$\Phi = \Phi\,(\theta, \frac{D}{d})\,. \tag{4.1.6}$$

Similarly, we get for the friction factor:

$$f = f(\theta, \frac{D}{d}) \tag{4.1.7}$$

Suppose that the ratio D/d is eliminated from the eqs. (4. 1. 6) and (4. 1. 7). We then get the alternative expression

$$\Phi = \Phi\,(\theta, f)\quad , \tag{4.1.8}$$

which is precisely the type of transport equation arrived at in section 4. 3.

4.2. Hydraulic Resistance of Alluvial Streams.

The rules of similarity discussed in the previous section were based on the consideration of two streams of different scale but with identical slopes. It is hardly possible to proceed much further without introducing additional assumptions or empirical results.

From the theory of model analysis it is well-known that rigorous scaling may be possible even when the vertical length scale differs from the horizontal. In such cases we talk about models with "distorted vertical scale". This concept of distortion plays an important role when we proceed to compare two streams 1 and 2 of different slopes, figure 4.2.1.

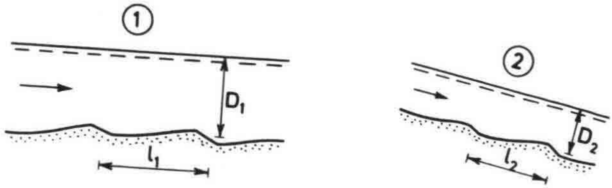

Figure 4.2.1

In this case similarity is possible (in the sense mentioned) when the following conditions are fulfilled:

1) The effective bed shear stresses are equal ($\theta'_1 = \theta'_2$) expressing dynamic similarity.

2) The expansion loss must be the same fraction of the total energy loss, so that the total bed shear is divided at the same proportion in both cases. The second condition may be elucidated if we apply eq. (4.1.5) on both streams:

$$f_1 = f'_1 + \frac{\alpha_1 h_1^2}{D_1 l_1} \text{ and } f_2 = f'_2 + \frac{\alpha_2 h_2^2}{D_2 l_2} .$$

Then 2) may be expressed in mathematical terms as the equation

$$\frac{f_1}{f_2} = \frac{f'_1}{f'_2} = \frac{\lambda_H}{\lambda_L} , \tag{4.2.1}$$

in which λ_H is the vertical length scale ratio ($\lambda_H = D_1/D_2$), while λ_L is the horizontal.

Alternatively, eq. (4.2.1) may be written

$$\frac{l_1 f_1}{h_1} = \frac{l_2 f_2}{h_2} \tag{4.2.2}$$

and correspondingly for f'.

Now we make the assumption (working hypothesis) that alluvial streams tend to adjust their bed roughness according to the outlined rules of similarity with distorted vertical scale. The exact meaning of this assumption is as follows:

In two streams with the same value of θ' the dune formations will be similar (with distortion) to such an extent that eq. (4.2.1) must hold.

As

$$\frac{f'}{f} = \frac{\theta'}{\theta}$$

we conclude that the parameter θ must be common too, so that

$$\theta = \theta(\theta') \tag{4.2.3}$$

This consequence is tested by adaptation of the extensive flume experiments carried out at Fort Collins by H. P. Guy, D. B. Simons and E. V. Richardson,[11]. The experiments are performed in an 8 ft wide flume, the grain sizes being d = 0.19, 0.27, 0.45 and 0.93 mm. For these experiments θ' was calculated and plotted against θ, figure 4.2.2.

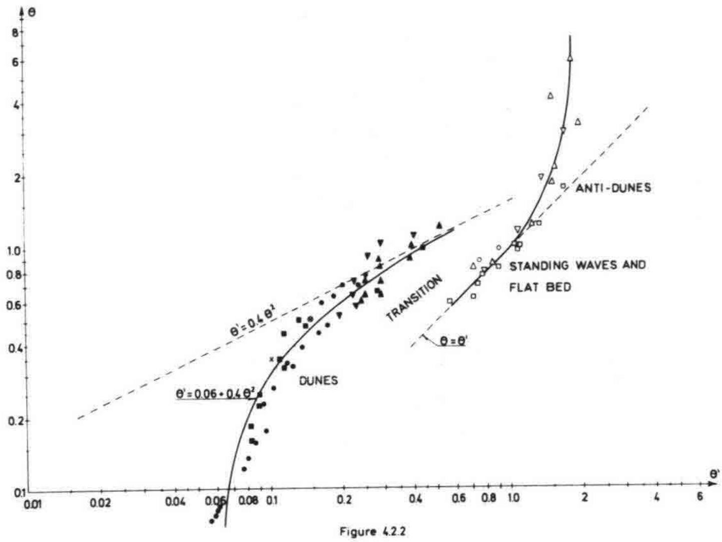

Figure 4.2.2

The points corresponding to dunes (lower flow regime) indicate a relationship between θ' and θ that may be approximated by the equation

$$\theta' = 0.06 + 0.4\,\theta^2 \tag{4.2.4}$$

For greater values of θ ($\theta > 0.15$) the asymptotic expression

$$\theta' = 0.4\,\theta^2 \tag{4.2.5}$$

will often suffice. This was found previously by a principle of similarity, [20] ~~[14]~~.

Note, that for decreasing values of θ, θ' approaches the threshold value 0.06 corresponding to the critical bed shear τ_c, see eq. (3.2.5)

Points corresponding to the upper flow regime (plane bed, standing waves, anti-dunes) constitutes a separate curve. To avoid confusion the points corresponding to the transition between upper and lower flow regime are omitted. In case of plane bed and standing waves no expansion losses are taking place. Consequently, we find $\theta = \theta'$.

The diagram in figure 4.2.2 is actually a universal, non-dimensional edition of the plot indicated in figure 3.4.1. This is realized if we calculate the parameter

$$\theta' = \frac{D' \, I}{1.65d}$$

by the substitution of D' from the approximate expression (3.6.8):

$$\theta' = \frac{I^{0.2}}{50} \left(\frac{V}{\sqrt{g\,d}}\right)^{1.6} \qquad (4.2.6)$$

The parameter θ is recalled to be

$$\theta = \frac{\tau_0}{\gamma(s-1)\,d} \quad , \qquad (4.2.7)$$

so that a relationship between θ' and θ constitutes in reality a relationship between V and τ_0.

The diagram in figure 4.2.2 may be said to test the validity of the similarity assumption. Another consequence of the assumption is, that the quantity fl/h should depend on θ' (or θ) only. By adaptation of the flume experiments it was found to be constant with the average value

$$\frac{fl}{h} \sim 0.47 \,, \qquad (4.2.8)$$

and a standard deviation of about 35 pct. Considering the irregularity and stochactic nature of the dune configuration that makes a concise definition of the dimensions difficult, this relatively large scatter is to be considered quite acceptable. For the lower flow regime (dunes) eq. (4.2.4) permits prediction of the hydraulic resistance. To see this we concentrate for a while on the simpl e asymptotic expression (4.2.5), which is sufficiently accurate for the majority of practical cases.

By substitution of eqs. (4.2.6) and (4.2.7) we obtain the simple power formula

$$V = K\,D^{5/4}\,I^{9/8} \quad , \quad \text{where } K = \frac{10.9}{d^{3/4}} \tag{4.2.9}$$

This is an expression of the same type as the conventional Manning formula, commonly used in hydraulic engineering. The factor 10.9 assumes $s = 2.65$ and the application of metric units, [14].

Eq. (4.2.9) has been found to give a satisfactory prediction of the stage-discharge relations of some American rivers, [21].

The problem of predicting the hydraulic resistance of alluvial streams has been treated previously by several investigators. Some of the first and most successful are H. A. Einstein and N. L. Barbarossa, [22]. They based their analysis on a large number of observations from American rivers. The same observations were included in the analysis performed in [14]. Thus eq. (4.2.9) is also known to be in accordance with these measurements.

4.3. Sediment discharge.

To complete the theory we still need a method for determination of the sediment discharge that is the total volume of moving sand particles per unit time. In the argumentation used to derive a transport formula we again concentrate on the dune case, and obtain results of more general applicability as originally found in [30]. In the deduction the results from the previous section are utilized.

The starting point is the simple transport mechanism outlined in section 3.4, according to which grains are eroded from the stream side of the dunes and are deposited at the lee side. Consequently along the stretch l the moving sediment load q_T is elevated a height comparable to the dune height h. This process may be described roughly by means of a simple energy equation.

The energy (per unit time and width) required to elevate the sediment discharge q_T a height of the order of magnitude h amounts to

$$(\gamma_s - \gamma)\, q_T\, h.$$

This gain in potential energy must be equal to the work done by the drag forces on the moving particles in the same time interval. According to Bagnold's theory (section 3. 3) the shear stress $\tau' - \tau_c$ is the stress transferred from the fluid to the moving sediment particles. The average migration velocity of the particles is assumed to be proportional to the friction velocity. Hence, we get the equation

$$(\gamma_s - \gamma)\, q_T\, h = \alpha(\tau' - \tau_0)\, l\, U_f \, , \tag{4.3.1}$$

in which α is a non-dimensional coefficient. By rearrangement we get

$$f\, q_T \left(\frac{h}{f\, l}\right) = \alpha \frac{\tau' - \tau_c}{(\gamma_s - \gamma) d}\, U_f\, d$$

By substitution of eq. (3. 2. 3):

$$f\, q_T \left(\frac{h}{f\, l}\right) = \alpha\, (\theta' - 0.06)\sqrt{\theta}\sqrt{(s-1) g\, d^3}$$

Now we introduce the non-dimensional sediment discharge defined by

$$\Phi = \frac{q_T}{\sqrt{(s-1) g\, d^3}} \, , \tag{4.3.2}$$

(see section 3. 3) and recall from section 4. 2 that $f\, l/h$ is a constant:

$$f\, \Phi \sim (\theta' - 0.06)\sqrt{\theta} \tag{4.3.3}$$

From eq. (4. 2. 4) we get

$$\theta' - 0,06 = 0.4\, \theta^2 \, ,$$

so that the transport equation becomes

$$f\, \Phi \sim \theta^{5/2} \tag{4.3.4}$$

The validity of a transport formula of this type is tested by comparison with the Fort Collins data utilized in the previous section, see figure 4. 3. 1. It is remarkable that the experiments comprise dunes, transition, standing waves, anti-dunes and chute-and-pool flow. The straight line indicated in the figure corresponds to the equation

$$f\, \Phi = 0.1\, \theta^{5/2} \tag{4.3.5}$$

in perfect agreement with the theoretical result.

It may be inferred that it is more logical to assume the mean particle velocity to be proportional to a friction velocity.

$$U'_f = \sqrt{\tau'/\rho} \, ,$$

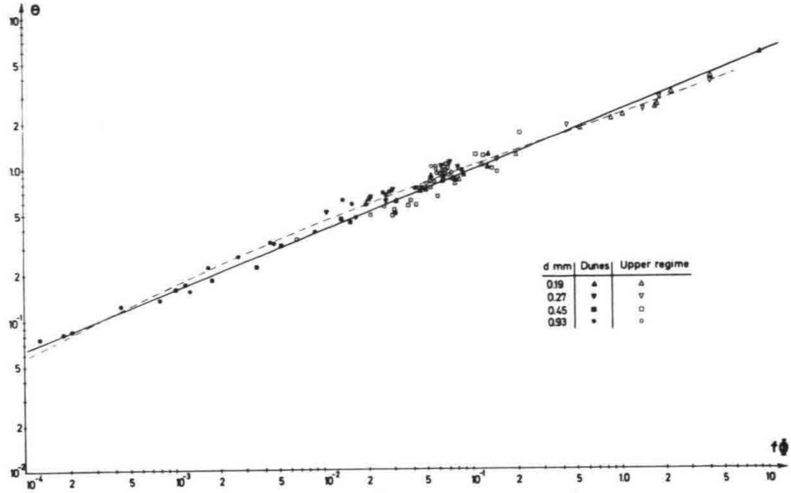

Figure 4.3.1

based on the effective shear. Accepting this we obtain by application of the same arguments

$$f\,\Phi \sim (\theta' - 0.06)\sqrt{\theta'}$$

instead of eq. (4.3.3).

This leads to a transport of slightly different appearance:

$$f\,\Phi = 0.077\,\theta^2\sqrt{\theta^2 + 0.15} \qquad (4.3.6)$$

indicated by the dotted line in figure 4.3.1. It is seen to fit the points as well as the straight line.

For small values of θ eq. (4.3.6) approaches an asymptotic relation of the type

$$f\,\Phi \sim \theta^2 \qquad (4.3.7)$$

For large values of θ, on the contrary, we obtain another asymptotic expression:

$$f\,\Phi \sim \theta^3$$

The second power relation corresponds to bed load transportation only, while the third power relation indicates a dominating transport of suspended particles.

The present experiments do not permit a definite choice between the two transport equations (4.3.5) and (4.3.6). As eq. (4.3.5) is more algebraic convenient it has been applied in the following.

4.4. Limitation of the theory.

Now we have arrived at the difficult problem of judging the range of validity of the theory presented. This is to a large extent a matter of estimating its accuracy, and it is most probable that much research has to be carried out before this question may be answered in a satisfactory way.

A few preliminary indications should be given here. However, the authors want to point out that these indications should be taken with much reservation.

The authors have found no serious deviations from the theory for sediments with sorting coefficients below about 1.6 and mean fall diameters larger than 0.15 mm. For very graded sediments containing abnormally large amounts of the finer fractions the actual sediment discharge seems to be essentially larger than predicted, at least for the smaller transport rates.

Reservations must be taken, when the sediment size is smaller than about 0.15 mm. This may be due to the fact that very large suspension seems to change the velocity distribution seriously, so that similarity becomes impossible.

5. FLUVIOLOGY.

5.1. General aspects.

From a geological point of view no rivers can be considered in exact equilibrium as they always take part in the disintegration process of the continental shelf and consequently change their courses continuosly.

For practical engineering purposes, however, it is often realistic to accept streams as being in equilibrium, if they have not changed their characteristics notably in a number of years, seasonal changes being disregarded.

As mentioned in the introduction there are several sources of the sediment transported in alluvial streams. For "equilibrium" streams, however, that resulting from sheet erosion as a result of surface runoff is usually by far the most important.

Thus it follows that the sediment discharge of a river primarily is determined by the nature of the drainage area. From hydrology the same is known to be true for the water discharge. Or stated in another way: Both sediment and water discharge are factors essentially independent of the river itself and almost exclusively determined by the hydrology, geology and topography of the drainage area.

For a given water and sediment discharge the river itself will now tend to create the geometry of its stream channel in a specific manner. This means that if the river is left alone for sufficiently long time with fixed values of the water and sediment discharge it will adjust itself with a definite slope, depth, width and meander pattern.

Since such factors as the slope and the meander pattern do not respond rapidly enough to follow the seasonal variations of the discharges it is natural to introduce some kind of "dominant", or "formative" values of the discharges with respect to these features. The "dominant discharge" should then be defined as the steady discharge which gives rise to the same slope and meander pattern as the annual sequence of discharges. As the larger discharges must be those responsible for the creation of the slope and the meander pattern, another - more practical - but also more arbitrary definition of the dominant discharge would be the discharge that is only exceeded e.g. 25 pct of the time.

For a given river it is normally found that the water and sediment discharges increase in the downstream direction. The same is found to be the case for the depth and the width of the stream. The slope and the grain size usually decrease gradually from souce to estuary. It is found, [23], that the grain size decreases approximately exponentially in the downstream direction.

These observations makes it natural to look for empirical relations for the variation of the depth, width and slope as functions of the water discharge, which quantity has turned out to be the most important independent parameter. This has really been done and quite a large number of "regime" relations has been suggested by various scientists. A recent account to these relations has been given by T. Blench, [24] and [25].

$$B \sim Q^{1/2} \tag{5.1.1}$$

$$D \sim Q^{1/3} \tag{5.1.2}$$

$$I \sim Q^{-1/6} \tag{5.1.3}$$

where B denotes the width, D the depth, I the slope and Q the water discharge. It has to be pointed out that in the present edition of Blench's formulae all variations with sediment discharge and grain size have been omitted for the sake of simplicity. In section 5.2 more elaborate forms of Blench's relations will be quoted.

According to Blench, rivers with erodible banks always tend to meander or at least create alternate shoals in apparently straight-looking reaches. These observations agree excellently with theoretical results, [26]. In this paper is demonstrated that if the formation of meanders is investigated as a usual hydrodynamic stability problem, meanders or shoaling will always occur in alluvial streams having dunes on the bed.

An empirical relation expressing the observations of meander "wave-lengths" has been suggested by C.C. Inglis, [27].

$$L \sim Q^{1/2} \tag{5.1.4}$$

Comparison of eqs. (5.1.1) and (5.1.4) indicates a direct proportionality between the width of the stream and the meander length. This was investigated empirically by L.B. Leopold and M.G. Wolman, [28], for a large number of American and Indian rivers as well as for several small scale model tests. From their data the following relation is found to be valid over several decades of the widths.

$$L = 10\,B \tag{5.1.5}$$

Taking into consideration the subjectiveness of assessment of meander lengths and stream width as well as the disturbing influence of local geological formations the observed deviations from eq. (5.1.5) are surprisingly small.

5.2 Application of the principle of similarity.

In this section we will deduce (on the basis of the principle of similarity) formulae for the depth, width, slope and meander length as functions of the water and sediment discharges as well as the grain size.

For this purpose we draw at first attention to eqs. (4.2.9) and (4.3.5) expressing the hydraulic resistance and the transport capacity of alluvial streams, respectively.

$$V = 10.9\, d^{-3/4} D^{5/4} I^{9/8} \tag{4.2.9}$$

$$f\, \Phi = 0.1\, \theta^{5/2} \tag{4.3.5}$$

(Note: While the last expression has broad applicability, the first is valid for the dune case only, and for $\theta > 0.15$).

Utilizing that L as well as B is a horizontal measure while D is a vertical measure we obtain by the use of eqs. (4.2.1) and (4.2.8) the following relationship.

$$f\,\frac{L}{D} = c_1 \qquad\qquad f\,\frac{B}{D} = c_2 \tag{5.2.1}$$

where c_1 and c_2 are constants. From these relations it is obvious that the principle of similarity predicts a direct proportionality between L and B. As mentioned in section 5.1 such a proportionality is really confirmed empirically, and the appropriate constant of proportionality was found to be

$$L/B = 10 \tag{5.1.5}$$

Concerning the magnitude of the constants c_1 and c_2 in eq. (5.2.1) the principle of similarity gives, however, no information.

In this connection it is interesting to note that in the stability analysis previously mentioned, [26], the following expressions for the meander length is obtained:

$$f\,\frac{L}{D} = 14 \tag{5.2.2}$$

that is exactly a result of the form predicted by the principle of similarity and further supplying a specific value 14 for the constant c_1, and thus by eq. (5.1.5) $c_2 = 1.4$.

Hence, by elementary calculations and by utilizing the definition expressions for f, θ and Φ along with eqs. (4. 2. 9), (4. 3. 5), (5. 1. 5) and (5. 2. 2) the following expressions are derived:

$$B = 0.78\, d^{-0.316}\, Q^{0.525} \tag{5.2.3}$$

$$D = 0.108\, (Q_T/Q)^{-2/7}\, d^{0.21}\, Q^{0.317} \tag{5.2.4}$$

$$I = 12.8\, (Q_T/Q)^{4/7}\, d^{0.527}\, Q^{-0.212} \tag{5.2.5}$$

$$L = 7.8\, d^{-0.316}\, Q^{0.525} \tag{5.2.6}$$

Because of the limited validity of eq. (4. 2. 8) these equations can only be considered valid for $\theta > 0.15$. In practice this is, however, no severe restriction.

Stating the results of the similarity investigations in the form of eqs. (5. 2.3)-(5. 2.6) it becomes easier to compare these results with the complete empirical findings, presented in the form of regime formulae. Such a comparison is made in table 5. 1 considering only the case of small sediment discharges for which Blench's formulae, [25], may be stated as shown in the table.

Table 5. 1.

	Variation with water discharge Q and grain size d.	
	Principle of similarity.	Regime formulae
Width, B	$d^{-0.316}\ Q^{0.525}$	$d^{-1/4}\ Q^{1/2}$
Depth, D	$d^{0.21}\ Q^{0.317}$	$Q^{1/3}$
Slope, I	$d^{0.527}\ Q^{-0.212}$	$d^{1/2}\ Q^{-1/6}$

Considering the table it becomes obvious that the variations with the water discharge are nearly identical in the two columns. With respect to the variation with the grain size greater deviations are noticed. However, in the only case, where the grain size is a really important factor, (with respect to the slope) the agreement is satisfactory.

Regarding the influence of the sediment discharge a similar comparison could be made. This has been done in [29]. The result of such an investigation indicates a satisfactory agreement with respect to most clarified cases, viz. those for the depth and slope. For the width, however, there is some deviation between the similarity result and the corresponding regime formula.

6. NUMERICAL EXAMPLES.

6.1. Prediction of stage-discharge relations.

To illustrate the practical application of the formulae developed in the preceeding sections we now give a couple of practical examples. First the formulae are applied to the problem of predicting stage discharge relations for rivers.

Example 6.1.1.

We consider the data from Pigeon Roost Creek (near Byhalia), [21]:

$$\text{Slope } I = 9 \cdot 10^{-4}$$

$$s = 2.68$$

The grain sizes from sieve analysis are given as

$$d_{35} = 0.35 \text{ mm} \quad \text{and} \quad d_{65} = 0.45 \text{ mm}$$

From table 2.2.1 the mean fall velocity is estimated to be $d = 0.38$ mm.

The procedure is then the following:

We choose a mean water depth, f.i. $D = 0.30$ m, and calculate the corresponding discharge in the following way:

From eq. (3.2.3):

$$\theta = \frac{D\,I}{(s-1)\,d} = \frac{0.3 \cdot 9 \cdot 10^{-4}}{1.68 \cdot 0.38 \cdot 10^{-3}} = 0.422$$

Next, from eq. (4.2.4):

$$\theta' = 0.06 + 0.4\,\theta^2 = 0.132$$

From this we get

$$D' = \frac{\theta'}{\theta}\,D = \frac{0.132}{0.422}\,0.3 = 0.094 \text{ m.}$$

Then the mean velocity V is found from eq. (3.6.6). The surface roughness is $k = 2.5\,d = 0.95$ mm.

$$\frac{V}{\sqrt{g D' I}} = 0.6 + 2.5 \ln \frac{D'}{k} \qquad (3.6.6)$$

$$V = \sqrt{9.8 \cdot 0.094 \cdot 9 \cdot 10^{-4}} \left(6.0 + 2.5 \ln \frac{94}{0.95}\right) = 0.50 \text{ m/sec}$$

The discharge q per unit width becomes

$$q = V D = 0.50 \cdot 0.30 = 0.15 \text{ m}^2/\text{sec}$$

The total sediment discharge may then be calculated from eq. (4.3.5):

$$f \Phi = 0.1 \, \theta^{5/2} \qquad (4.3.5)$$

The friction factor f is obtained from the expression

$$f = \frac{2 I}{\mathbf{F}^2} = \frac{2g D I}{V^2} = \frac{2 \cdot 9.8 \cdot 0.30 \cdot 9 \cdot 10^{-4}}{0.50^2} = 0.021$$

Hence we get:

$$\Phi = \frac{0.1}{0.021} \, 0.422^{5/2} = 0.55$$

Then the total sediment discharge q_T is found to be

$$q_T = \Phi \sqrt{(s-1) g \, d^3} = 0.55 \cdot 0.3 \cdot 10^{-4} = 1.65 \cdot 10^{-5} \text{ m}^2/\text{sec}$$

Finally we have to make sure that the flow actually belongs to the dune regime. To this end we calculate

$$U_f = V \sqrt{\tfrac{1}{2} f} = 0.50 \cdot \sqrt{0.0105} = 0.051 \text{ m/sec}$$

Next we form Reynolds' number

$$\mathbf{R} = U_f \, d/\nu = 0.051 \cdot 0.38 \cdot 10^{-3} / 1.3 \cdot 10^{-6} = 15.$$

As $\mathbf{R} > 12$ we assume that the bed form is actually dunes rather than ripples.

Example 6.1.2.

Sufficient accuracy is usually obtained by the application of the design chart at the end of the book. As an example let us solve the problem from example 6.1.1 by means of this chart.

First we calculate the quantity

$$D/d = 0.30/0.38 \cdot 10^{-3} = 790$$

Next we find the point for which

$$D/d = 790 \quad \text{and} \quad I = 9 \cdot 10^{-4}$$

The ordinate of this point is

$$\frac{V\ D}{\sqrt{(s-1)\ g\ d^3}} = 5 \cdot 10^3$$

from which

$$q = V\ D = 0.30 \cdot 10^{-4} \cdot 5 \cdot 10^3 = 0.15\ m^2/sec$$

The abscissa of the point is

$$\Phi = 0.55.$$

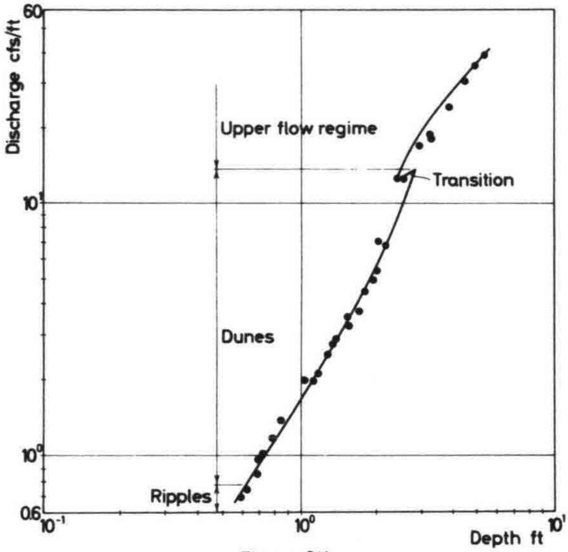

Figure 6.1.1

In the example only a single point of the stage-discharge relation was found. In order to obtain a complete stage-discharge curve - as that indicated in figure 6. 1. 1 - the procedure must be repeated for a sufficient number of depths.

Besides its convenience the design chart has the advantage of indicating the flow regime. The transition between the lower and the upper regime is assumed to occur abruptly, when the values of D/d and I correspond to a point situated at the left hand boundary of the white strip in the design chart. (For the river treated in the preceding example we find the transition value of $D/d \simeq 2700$). For increasing velocity the points will now correspond to the upper flow regime, and the upper curve of figure 4.2.2 must then be applied for the prediction of the stage-discharge relation instead of eq. (4.2.4).

In this way the relation is obtained for Pigeon Roost Creek as given by full line in figure 6.1.1. The calculation is compared with actual measurements as reported in [21].

6.2. Design of channels.

As previously stated (section 3.4) Q, Q_T and d are in fact more fundamental variables than those used by the theoretical deductions made in section 4, (D, I and d). This corresponds to the fact that normally Q and Q_T may be considered the real independent variables, while D and I correspondingly are dependent variables. The basic nature is illustrated by the fact that the flow regime (except with regard to ripples) is found to be uniquely determined by the variables Q, Q_T and d in the upper as well as in the lower flow regime.

For this reason it is natural to formulate the results obtained in section 4 in terms of the more fundamental variables.

This has been done by the construction of a design chart as shown in figure 6.2.1 at the end of the book. It should be pointed out that the chart is not valid for the case of ripples, when $U_f\, d/\nu < 12$. At the preparation, the following formulae were utilized

$$\frac{V}{\sqrt{g\, D'\, I}} = 6.0 + 2.5 \ln \frac{D'}{2.5\, d} \qquad (3.6.6)$$

$$\theta' = 0.06 + 0.4\, \theta^2 \qquad (4.2.4)$$

$$f\, \Phi = 0.1\, \theta^{5/2} \qquad (4.3.5)$$

In order to determine the region, expressed in the parameters $V\, D/\sqrt{g(s-1)\, d^3}$ and Φ, within which dunes are the dominant bed configurations (and the design chart applicable), the experiments previously mentioned [11], were plotted in a $(V\, D/\sqrt{g(s-1)\, d^3}, \Phi)$-diagram. The location of the various flow regimes as obtained in this way is shown in figure 6.2.1 except with regard to ripples, for which case viscous effects intervene. The result may be shown to agree in principle with the results obtained by stability analysis [14].

Within the region where dunes occur the relevant parameter-curves for D/d and I are drawn.

If, for a given bed material, the water and sediment discharge per unit width of the stream is known, the corresponding values of depth and slope are immediately obtainable from figure 6.2.1.

Considering another situation: if, for a given bed material, the slope and the water discharge per unit width is known, the corresponding values of depth and sediment discharge per unit width are quite as easily obtained from the diagram.

Thus figure 6.2.1 is well suited for design purposes.

The procedure is illustrated by a numerical example.

Example 6.2.2.

If an alluvial channel of width 10 m has to carry a water discharge $Q = 6.0\ m^3/s$ and a sediment discharge $Q_T = 2.6 \cdot 10^{-4}\ m^3/s$ what will be the corresponding natural depth and slope? The grain size of the sediment is $d = 0.4$ mm.

At first we calculate the parameters

$$V D/\sqrt{g(s-1)\, d^3} = \frac{6.0}{10 \cdot 3.32 \cdot 10^{-5}} = 1.81 \cdot 10^4$$

$$\Phi = \frac{2.6 \cdot 10^{-4}}{10 \cdot 3.32 \cdot 10^{-5}} = 0.78$$

Next, by the application of the design chart, we find

$$I = 3.0 \cdot 10^{-4} \qquad \frac{D}{d} = 2.4 \cdot 10^3$$

Substitution of $d = 0.4$ mm yields for the depth

$$D = 0.96\ m.$$

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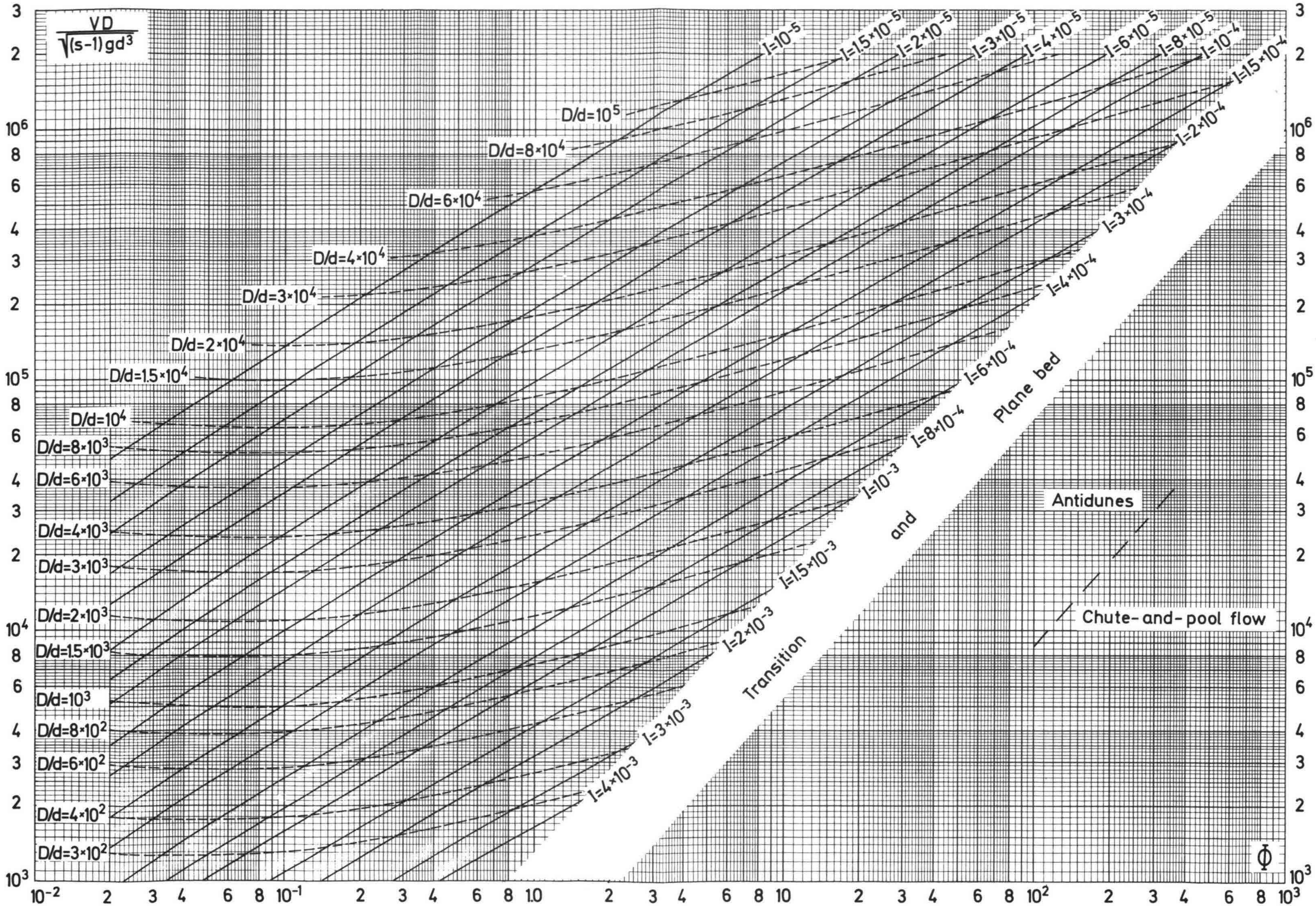

$\frac{VD}{\sqrt{(s-1)gd^3}}$
I=10-5
I=1.5×10-5
I=2×10-5
I=3×10-5
I=4×10-5
I=6×10-5
I=8×10-5
I=10-4
I=1.5×10-4
I=2×10-4
I=3×10-4
I=4×10-4
I=6×10-4
I=8×10-4
I=10-3
I=1.5×10-3
I=2×10-3
I=3×10-3
I=4×10-3
D/d=10^5
D/d=8×10^4
D/d=6×10^4
D/d=4×10^4
D/d=3×10^4
D/d=2×10^4
D/d=1.5×10^4
D/d=10^4
D/d=8×10^3
D/d=6×10^3
D/d=4×10^3
D/d=3×10^3
D/d=2×10^3
D/d=1.5×10^3
D/d=10^3
D/d=8×10^2
D/d=6×10^2
D/d=4×10^2
D/d=3×10^2
Transition and Plane bed
Antidunes
Chute-and-pool flow
Φ

Figure 6.2.1