

Elimination of quantifiers and undecidability in spatial logics for concurrency

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Abstract

The introduction of spatial logics in concurrency is motivated by a shift of focus from concurrent systems towards distributed systems. Aiming at a deeper understanding of the essence of dynamic spatial logics, we study a minimal spatial logic without quantifiers or any operators talking about names. The logic just includes the basic spatial operators void, composition and its adjunct, and the next step modality; for the model we consider a tiny fragment of CCS. We show that this core logic can already encode its own extension with quantification over actions, and modalities for actions. From this result, we derive several consequences. Firstly, we establish the intensionality of the logic, we characterize the equivalence it induces on processes, and we derive characteristic formulas. Secondly, we show that, unlike in static spatial logics, the composition adjunct adds to the expressiveness of the logic, so that adjunct elimination is not possible for dynamic spatial logics, even quantifier-free. Finally, we prove that both model-checking and satisfiability problems are undecidable in our logic. We also conclude that our results extend to other calculi, namely the π -calculus and the ambient calculus.

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0. Introduction

The introduction of spatial logics in concurrency has been motivated by a recent shift of focus from monolithic concurrent systems towards distributed computing systems. Such systems are by nature both concurrent and spatially distributed, in the sense that they are composed from a number of separate and independently observable units of behavior and computation. The central idea behind spatial logics is that for specifying distributed computations there is a need to talk in a precise way not just about pure behaviors, as is the case with traditional logics for concurrency, but about a richer model able to represent computation in a space. Such an increased degree of expressiveness is necessary if we want to specify with and reason about notions like locations, resources, independence, distribution, connectivity, and freshness. Spatial logics have been proposed for π -calculi [4,3], and for the ambient calculus [11,12]. Spatial logics for manipulating and querying semi-structured data have also been developed [9,8]. Closely related are the separation logics [21,20], introduced with the aim of supporting local reasoning about imperative programs.

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